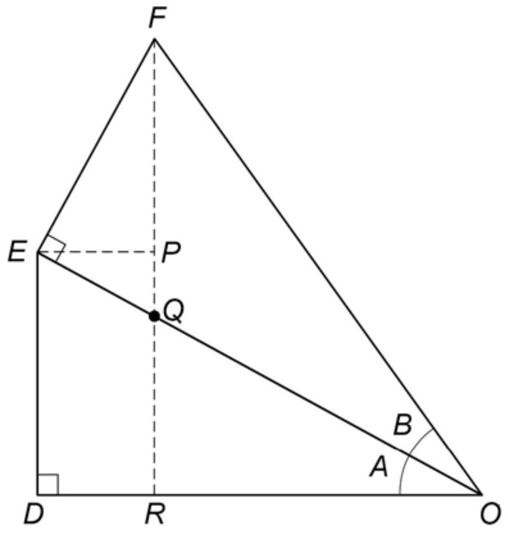
14 Some students are trying to prove an identity for $\sin(A+B)$.

They start by drawing two right-angled triangles *ODE* and *OEF*, as shown.



The students' incomplete proof continues,

Let angle $DOE = A$ and angle $EOF = B$.

In triangle *OFR*,

Line 1 $\sin(A+B) = \dfrac{RF}{OF}$

Line 2 $= \dfrac{RP+PF}{OF}$

Line 3 $= \dfrac{DE}{OF} + \dfrac{PF}{OF}$ since $DE = RP$

Line 4 $= \dfrac{DE}{.....} \times \dfrac{.....}{OF} + \dfrac{PF}{EF} \times \dfrac{EF}{OF}$

Line 5 $= ....+ \cos A \sin B$

14 (a) Explain why $\dfrac{PF}{EF} \times \dfrac{EF}{OF}$ in Line 4 leads to $\cos A \sin B$ in Line 5

[2 marks]

14 (b) Complete Line 4 and Line 5 to prove the identity

Line 4 $= \dfrac{DE}{.....} \times \dfrac{.....}{OF} + \dfrac{PF}{EF} \times \dfrac{EF}{OF}$

Line 5 $= .... + \cos A \sin B$

[1 mark]

14 (c) Explain why the argument used in part **(a)** only proves the identity when *A* and *B* are acute angles.

[1 mark]

14 (d) Another student claims that by replacing *B* with $-B$ in the identity for $\sin(A+B)$ it is possible to find an identity for $\sin(A-B)$.

Assuming the identity for $\sin(A+B)$ is correct for all values of *A* and *B*, prove a similar result for $\sin(A-B)$.

[3 marks]