

Q	Marking instructions	AO	Mark	Typical solution
16(a)	Chooses an appropriate technique to differentiate accept any evidence of product rule or quotient rule	3.1a	M1	$\frac{dy}{dx} = -e^{-x}(\sin x + \cos x) + e^{-x}(\cos x - \sin x)$ $= -2e^{-x} \sin x$
	Differentiates fully correctly	1.1b	A1	
	Obtains fully correct simplified answer.	1.1b	A1	
16(b)	Uses their result from (a) in the form of $Be^{-x} \sin x$ showing an understanding of the fundamental theorem of calculus Condone missing constant.	3.1a	M1	$\int (-2e^{-x} \sin x) dx = e^{-x}(\sin x + \cos x) + k$ $\therefore \int (e^{-x} \sin x) dx = -\frac{1}{2}e^{-x}(\sin x + \cos x) + c$
	Obtains $\frac{1}{B}e^{-x}(\sin x + \cos x)$	2.1	R1F	

16 (c)(i)	Writes the area as $\frac{1}{B} \left[e^{-x} (\sin x + \cos x) \right]_0^\pi$ Condone missing limits	3.1a	M1	$\int_0^\pi (e^{-x} \sin x) dx = -\frac{1}{2} \left[e^{-x} (\sin x + \cos x) \right]_0^\pi$ $= -\frac{1}{2} [e^{-\pi} (\sin \pi + \cos \pi) - e^0 (\sin 0 + \cos 0)]$ $= \frac{e^{-\pi} + 1}{2}$
	Deduces correct limits and substitutes correctly	2.2a	A1	
	Obtains correct exact value from correct answer in part(b) CSO	1.1b	A1	
16 (c)(ii)	Substitutes correct limits for A_2 Into their $\frac{1}{B} \left[e^{-x} (\sin x + \cos x) \right]_0^\pi$ Or Writes $A_2 = \pm \int_\pi^{2\pi} (e^{-x} \sin x) dx$ and uses the substitution $u = x - \pi$	1.1a	M1	$\int_\pi^{2\pi} (e^{-x} \sin x) dx = -\frac{1}{2} \left[e^{-x} (\sin x + \cos x) \right]_\pi^{2\pi}$ $= -\frac{e^{-\pi} + 1}{2} e^{-\pi}$ $\text{Area} = \frac{e^{-\pi} + 1}{2} e^{-\pi}$ $\frac{A_2}{A_1} = \frac{\frac{e^{-\pi} + 1}{2} e^{-\pi}}{\frac{e^{-\pi} + 1}{2}} = e^{-\pi}$
	Obtains correct exact area for $\pm A_2$ CSO Or Makes complete substitution $A_2 = \pm \int_0^\pi (e^{-(u+\pi)} \sin(u+\pi)) du$	1.1b	A1	
	Forms required ratio using their exact A1 and A2, may be unsimplified Or Extracts factor of $e^{-\pi}$ and uses $\sin(u+\pi) = -\sin u$ To obtain $A_2 = -e^{-\pi} \int_0^\pi e^{-u} \sin u du$	1.1a	M1	
	Completes rigorous argument, with correct limits and negatives handled correctly CSO	2.1	R1	
16 (c)(iii)	Deduces that the areas form a geometric series Accept any indication of this being geometric series	2.2a	B1	$\frac{a}{1-r} = \frac{e^{-\pi} + 1}{2} \times \frac{1}{1 - e^{-\pi}}$ $= \frac{1 + e^\pi}{2(e^\pi - 1)}$
	Uses $\frac{A_1}{1 - e^{-\pi}}$	3.1a	M1	
	Obtains a value for the geometric series first term using their (c)(i) Completes rigorous argument to achieve required result in correct form. CSO AG	1.1a 2.1	B1F R1	
Total			16	