16 (a) $y = e^{-x}(\sin x + \cos x)$

Find $\frac{dy}{dx}$

Simplify your answer.

[3 marks]

16 (b) Hence, show that

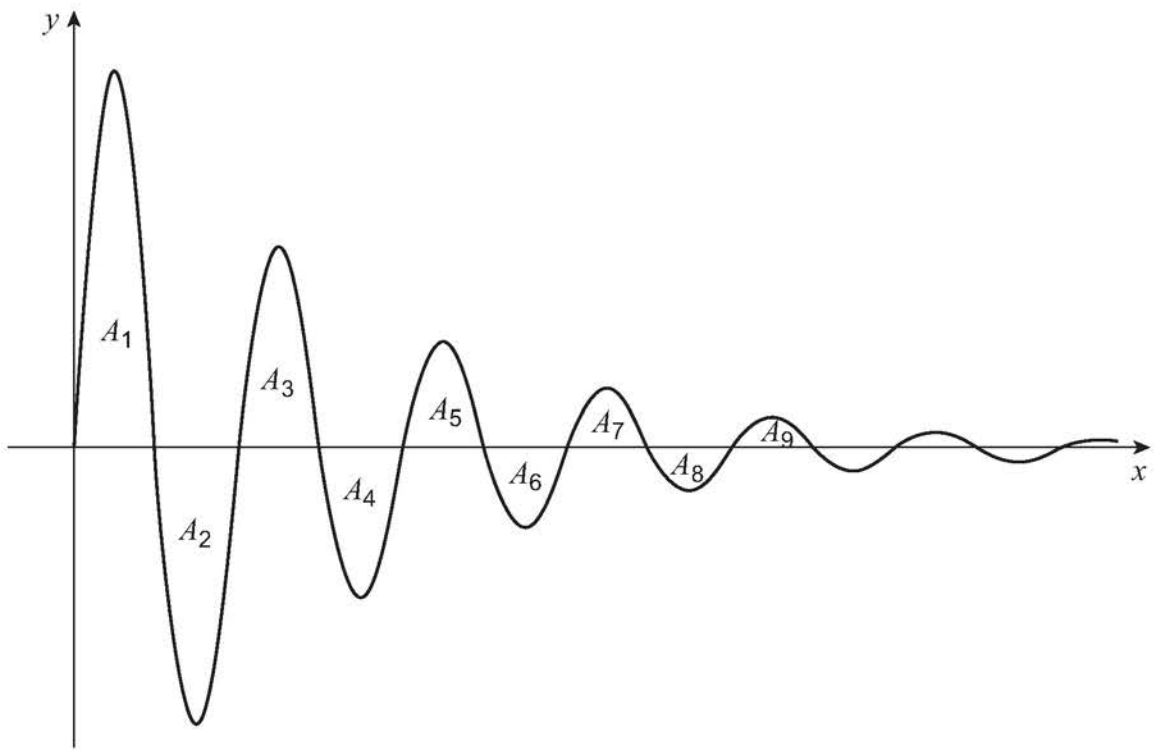
$$\int e^{-x} \sin x \, dx = ae^{-x}(\sin x + \cos x) + c$$

where a is a rational number.

[2 marks]

16 (c) A sketch of the graph of $y = e^{-x} \sin x$ for $x \geq 0$ is shown below.

The areas of the finite regions bounded by the curve and the x-axis are denoted by $A_1, A_2, \ldots, A_n, \ldots$



16 (c) (i) Find the exact value of the area A_1

[3 marks]

16 (c) (ii) Show that

$$\frac{A_2}{A_1} = e^{-\pi}$$

[4 marks]

16 (c) (iii) Given that

$$\frac{A_{n+1}}{A_n} = e^{-\pi}$$

show that the exact value of the total area enclosed between the curve and the x-axis is

$$\frac{1 + e^{\pi}}{2(e^{\pi} - 1)}$$

[4 marks]