

14 The function f is defined by

$$f(x) = 3^x \sqrt{x} - 1 \quad \text{where } x \geq 0$$

14 (a) $f(x) = 0$ has a single solution at the point $x = \alpha$

By considering a suitable change of sign, show that α lies between 0 and 1

[2 marks]

14 (b) (i) Show that

$$f'(x) = \frac{3^x(1 + x \ln 9)}{2\sqrt{x}}$$

[3 marks]

14 (b) (ii) Use the Newton–Raphson method with $x_1 = 1$ to find x_3 , an approximation for α .

Give your answer to five decimal places.

[2 marks]

14 (b) (iii) Explain why the Newton–Raphson method fails to find α with $x_1 = 0$

[2 marks]