Q	Marking instructions	AO	Marks	Typical solution
15(a)(i)	States $(\sin\theta)^{-1}$ or $\frac{1}{\sin\theta}$ $\sin^{-1}\theta$ scores B0 Ignore $\sin^{-1}\theta$ if a correct expression has already been written	1.2	B1	$\frac{1}{\sin\theta}$
	Subtotal		1	

Q	Marking instructions	AO	Marks	Typical solution
15(a)(ii)	Uses chain rule or quotient rule to obtain $\pm k(\sin\theta)^{-2}\cos\theta$ OE or Multiplies and uses product rule and implicit differentiation to obtain $\pm k(\sin\theta)^{-2}\cos\theta$ This mark can be awarded for using $\frac{1}{\cos\theta}$ and differentiating to obtain $\pm k(\cos\theta)^{-2}\sin\theta$ OE Ignore wrong or missing angles	3.1a	M1	
	Obtains $-(\sin\theta)^{-2}\cos\theta$ OE	1.1b	A1	$\frac{dy}{d\theta} = -(\sin\theta)^{-2}\cos\theta$ $= -\frac{\cos\theta}{\sin\theta}\times\frac{1}{\sin\theta}$ $= -\text{cosec}\,\theta\cot\theta$
	Completes **rigorous** argument to show the given result. Must either see separated fractions before final line or sight of $-\frac{\cot\theta}{\sin\theta}$ or $-\frac{1}{\tan\theta\sin\theta}$ or $-\frac{\cos\theta}{\sin\theta}\times\text{cosec}\,\theta$ or Makes clear use of stated identities as part of the solution At some point the solution must have included $\frac{dy}{d\theta} =$ AG Condone change of order of functions at the end	2.1	R1	
	Subtotal		3	

Q	Marking instructions	AO	Marks	Typical solution
15(a)(iii)	Substitutes $y = \text{cosec}\,\theta$ OE or Draws a right angled triangle labelling hypotenuse as y and opposite as 1 PI by obtaining y^2 or $\frac{1}{y^2}$ in terms of $\cos\theta$	1.1b	B1	$\frac{\sqrt{\text{cosec}^2\theta - 1}}{\text{cosec}\,\theta}$ $= \frac{\sqrt{\cot^2\theta}}{\text{cosec}\,\theta}$ $= \frac{\cot\theta}{\text{cosec}\,\theta}$ $= \frac{\cos\theta}{\sin\theta} \times \sin\theta$ $= \cos\theta$
	Uses $\text{cosec}^2\theta - 1 = \cot^2\theta$ OE or Uses Pythagoras theorem to find missing adjacent side in the right angled triangle or Obtains $\cos^2\theta$ in terms of y	1.1a	M1	
	Completes **rigorous** argument to show the given result This must include **clear replacement** of $\text{cosec}\,\theta$ and $\cot\theta$ **within the solution** using only sine and cosine functions to complete the argument prior to obtaining the answer given AG	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)(i)	Obtains $\frac{dx}{du} = -2\operatorname{cosec} u \cot u$ OE	1.1b	B1	$\frac{dx}{du} = -2\operatorname{cosec} u \cot u$ $dx = -2\operatorname{cosec} u \cot u \ du$ $= \int \frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u\sqrt{4\operatorname{cosec}^2 u - 4}} du$ $= \int \frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u\sqrt{4\cot^2 u}} du$ $= \int \frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u \times 2\cot u} du$ $= \int -\frac{1}{4\operatorname{cosec} u} du$ $= -\frac{1}{4}\int \sin u \ du$
	Makes complete substitution to obtain integrand of the form $\frac{P\operatorname{cosec} u \cot u}{Q\operatorname{cosec}^2 u\sqrt{R\operatorname{cosec}^2 u - 4}}$ OE or $\frac{P\operatorname{cosec} u \cot u}{Q\operatorname{cosec}^3 u\sqrt{1 - R\sin^2 u}}$ OE Ignore wrong or missing angles	1.1a	M1	
	Obtains correct integrand $\frac{-2\operatorname{cosec} u \cot u}{4\operatorname{cosec}^2 u\sqrt{4\operatorname{cosec}^2 u - 4}}$ or $\frac{-2\operatorname{cosec} u \cot u}{8\operatorname{cosec}^3 u\sqrt{1 - \sin^2 u}}$ OE	1.1b	A1	
	Uses appropriate Pythagorean-trig identity under the square root. Either $1 + \cot^2 u = \operatorname{cosec}^2 u$ or $1 - \sin^2 u = \cos^2 u$ Ignore wrong or missing angles	3.1a	M1	
	Obtains $k\int \sin u \ du$ with no errors seen in any trig identities Must have u and du	1.1b	A1F	
	Obtains $k = -\frac{1}{4}$ OE CSO	2.1	R1	
	Subtotal		**6**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)(ii)	Integrates $\int \sin u \ \mathrm{d}u$ to obtain $-\cos u$	1.1b	B1	$-\frac{1}{4}\int \sin u \ \mathrm{d}u = \frac{1}{4}\cos u + c$ $= \frac{1}{4}\frac{\sqrt{\left(\frac{x}{2}\right)^2 - 1}}{\left(\frac{x}{2}\right)} + c$ $= \frac{\sqrt{\frac{x^2-4}{4}}}{2x} + c$ $= \frac{\sqrt{x^2-4}}{4x} + c$
	Deduces $\cos u = \frac{\sqrt{\left(\frac{x}{2}\right)^2 - 1}}{\left(\frac{x}{2}\right)}$ OE	2.2a	M1	
	Completes **reasoned** argument to show given result Must have $+c$ throughout Validation by starting with $\frac{\sqrt{x^2-4}}{4x}$ and replacing x with $2\operatorname{cosec} u$ to achieve $\frac{1}{4}\cos u$ scores a maximum of B1M1R0	2.1	R1	
	Subtotal		**3**	

	Question 15 Total		**16**	