

15 (a) Given that

$$y = \operatorname{cosec} \theta$$

15 (a) (i) Express y in terms of $\sin \theta$.

[1 mark]

15 (a) (ii) Hence, prove that

$$\frac{dy}{d\theta} = -\operatorname{cosec} \theta \cot \theta$$

[3 marks]

15 (a) (iii) Show that

$$\frac{\sqrt{y^2 - 1}}{y} = \cos \theta \quad \text{for } 0 < \theta < \frac{\pi}{2}$$

[3 marks]

15 (b) (i) Use the substitution

$$x = 2 \operatorname{cosec} u$$

to show that

$$\int \frac{1}{x^2 \sqrt{x^2 - 4}} dx \quad \text{for } x > 2$$

can be written as

$$k \int \sin u \, du$$

where k is a constant to be found.

[6 marks]

15 (b) (ii) Hence, show

$$\int \frac{1}{x^2 \sqrt{x^2 - 4}} dx = \frac{\sqrt{x^2 - 4}}{4x} + c \quad \text{for } x > 2$$

where c is a constant.

[3 marks]