

| Q               | Marking instructions   | AO   | Marks    | Typical solution   |
|-----------------|--|------|----------|--|
| 16(a)           | Uses a suitable method and finds a value for $A$ or $B$ .<br>For example<br>Rearranges and substitutes values or compares coefficients<br>Or<br>Uses cover-up method<br>Or<br>Uses inspection<br>PI by $A$ correct or $B$ correct. | 1.1a | M1       | $\frac{1}{(4-3x)(4+3x)} \equiv \frac{A}{4-3x} + \frac{B}{4+3x}$ $1 \equiv A(4+3x) + B(4-3x)$ $\text{Let } x = -\frac{4}{3}$ $1 \equiv 8B \Rightarrow B = \frac{1}{8}$ $\text{comparing } x \text{ terms } A = B = \frac{1}{8}$ |
|                 | Obtains $A = \frac{1}{8}$  | 1.1b | A1       |  |
|                 | Obtains $B = \frac{1}{8}$  | 1.1b | A1       |  |
| <b>Subtotal</b> |  |      | <b>3</b> |  |

| Q               | Marking instructions   | AO   | Marks    | Typical solution  |
|-----------------|--|------|----------|---|
| 16(b)(i)        | Forms differential equation using $\frac{dV}{dt} = \pm 0.16 \pm 0.36d^2$   | 3.3  | M1       | $\frac{dV}{dt} = 0.16 - 0.36d^2$ $V = 1.25 \times 1.6d \Rightarrow d = \frac{V}{2}$ $\frac{dV}{dt} = 0.16 - 0.36 \left(\frac{V}{2}\right)^2$ $= 0.16 - 0.09V^2$ $= \frac{16 - 9V^2}{100}$ |
|                 | Obtains $V = 1.25 \times 1.6d$<br>OE   | 3.1b | B1       |   |
|                 | Substitutes their expression for $d$ into $\frac{dV}{dt} = \pm 0.16 \pm 0.36d^2$<br>to obtain a differential equation in $V$ and $t$ only. | 3.1a | M1       |   |
|                 | Completes reasoned argument to show the given result.<br>AG  | 2.1  | R1       |   |
| <b>Subtotal</b> |  |      | <b>4</b> |   |

| Q         | Marking instructions   | AO   | Marks    | Typical solution   |
|-----------|--|------|----------|--|
| 16(b)(ii) | <p>Rearranges to obtain one of the following:</p> $\frac{P}{16-9V^2} dV = \frac{1}{Q} dt$ $\frac{P}{16-9V^2} \frac{dV}{dt} = \frac{1}{Q}$ $\frac{P}{16-9V^2} = \frac{1}{Q} \frac{dt}{dV}$ <p>where <math>P \times Q = 100</math><br/>           If their <math>P = 100</math> no need to see <math>\frac{1}{Q}</math> explicit with <math>dt</math></p> <p>May include integral signs<br/>           PI <math>\int \frac{100}{16-9V^2} dV = t</math></p> | 3.1a | B1       | $\int \frac{1}{16-9V^2} dV = \int \frac{1}{100} dt$ $\frac{1}{8} \int \frac{1}{4-3V} + \frac{1}{4+3V} dV = \int \frac{1}{100} dt$ $\frac{1}{24} (-\ln(4-3V) + \ln(4+3V)) = \frac{t}{100} + c$ $t=0, V=0 \Rightarrow c=0$ $\frac{100}{24} (-\ln(4-3V) + \ln(4+3V)) = t$ |
|           | <p>Integrates their constant integrand correctly with respect to <math>t</math>.<br/>           Follow through any constant.</p>   | 1.1b | B1F      |  |
|           | <p>Writes <math>\int \frac{1}{16-9V^2} dV</math><br/>           as<br/> <math display="block">\int \frac{A}{4-3V} + \frac{B}{4+3V} dV</math><br/>           Condone missing <math>dV</math><br/>           PI by<br/> <math display="block">-\frac{A}{3} \ln(4-3V) + \frac{B}{3} \ln(4+3V)</math></p>  | 3.1a | M1       |  |
|           | <p>Integrates their partial fractions correctly to obtain<br/> <math display="block">-\frac{A}{3} \ln(4-3V) + \frac{B}{3} \ln(4+3V) \quad (+c)</math><br/>           OE<br/>           Their <math>A</math> and <math>B</math> may be correctly inside the natural logs for example<br/> <math display="block">\frac{1}{24} (-\ln(32-24V) + \ln(32+24V))</math></p>  | 1.1b | A1F      |  |
|           | <p>Completes argument, including demonstrating that the constant of integration is zero.</p>   | 2.1  | R1       |  |
|           | <b>Subtotal</b>  |      | <b>5</b> |  |

| Q          | Marking instructions   | AO   | Marks    | Typical solution  |
|------------|--|------|----------|---|
| 16(b)(iii) | Obtains a value for $t$ by substituting $V = 1$ into their expression for $t$ from their final answer from b(ii)<br>PI by 8 minutes from a correct answer from b(ii) | 3.4  | M1       | $t = \frac{100}{24}(-\ln(4-3) + \ln(4+3))$ $= \frac{100}{24} \ln 7$ $= 8 \text{ minutes}$ |
|            | Obtains 8 minutes following a correct answer in b(ii)<br>Condone missing units   | 3.2a | A1       |   |
|            | <b>Subtotal</b>  |      | <b>2</b> |   |

|  |                          |  |           |  |
|--|--------------------------|--|-----------|--|
|  | <b>Question 16 Total</b> |  | <b>14</b> |  |
|--|--------------------------|--|-----------|--|