

14 (a) (i) Given that

$$y = 2^x$$

write down  $\frac{dy}{dx}$

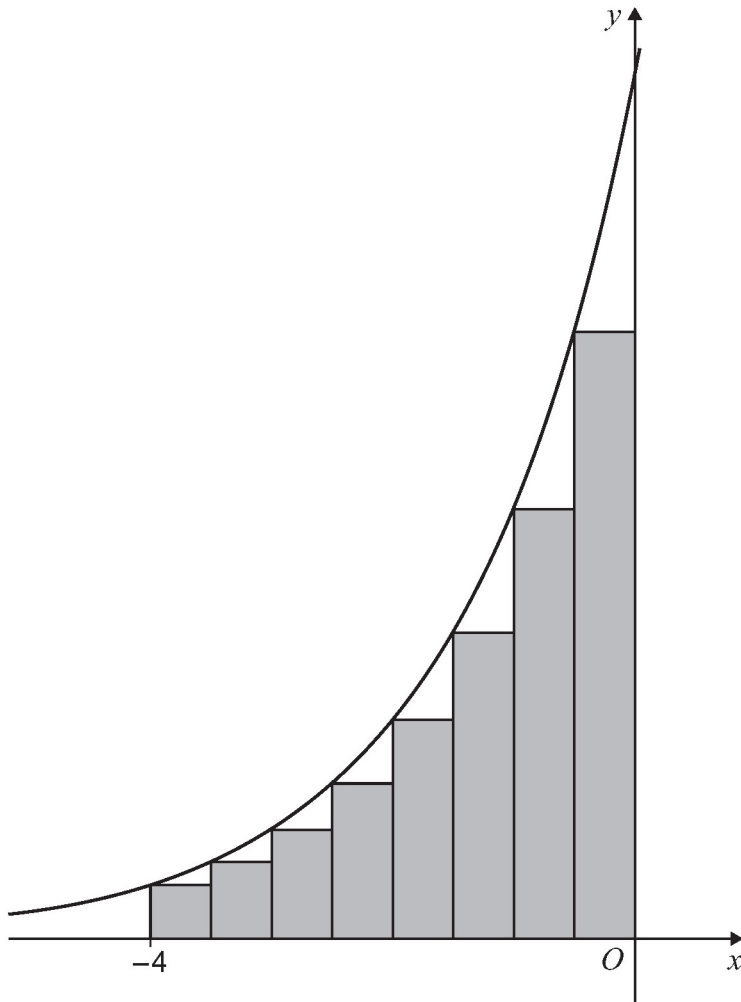
[1 mark]

14 (a) (ii) Hence find

$$\int 2^x dx$$

[2 marks]

14 (b) The area,  $A$ , bounded by the curve with equation  $y = 2^x$ , the  $x$ -axis, the  $y$ -axis and the line  $x = -4$  is approximated using eight rectangles of equal width as shown in the diagram below.



14 (b) (i) Show that the exact area of the largest rectangle is  $\frac{\sqrt{2}}{4}$

[2 marks]

14 (b) (ii) The areas of these rectangles form a geometric sequence with common ratio  $\frac{\sqrt{2}}{2}$

Find the exact value of the total area of the eight rectangles.

Give your answer in the form  $k(1 + \sqrt{2})$  where  $k$  is a rational number.

[3 marks]

14 (b) (iii) More accurate approximations for  $A$  can be found by increasing the number,  $n$ , of rectangles used.

Find the exact value of the limit of the approximations for  $A$  as  $n \rightarrow \infty$

[3 marks]