Q	Marking instructions	AO	Marks	Typical solution
18(a)	Selects the substitution $u=2x+1$ and differentiates or uses it to replace $2x+1$ in the integrand.	3.1a	B1	Let $u=2x+1$ $\frac{\mathrm{d}u}{\mathrm{d}x}=2 \Rightarrow dx=\frac{1}{2}du$ $4x+1=2u-1$ $\int_0^4 (4x+1)(2x+1)^{\frac{1}{2}}\,\mathrm{d}x$ $=\int_1^9 (2u-1)(u)^{\frac{1}{2}}\frac{1}{2}\mathrm{d}u$ $=\frac{1}{2}\int_1^9 \left(2u^{\frac{3}{2}}-u^{\frac{1}{2}}\right)\mathrm{d}u$
	Differentiates their substitution and uses the result to replace $\mathrm{d}x$ in the integral.	1.1a	M1	
	Makes a complete substitution to write the integrand in terms of u leading to an integrand of the form $A(2u-k)u^{\frac{1}{2}}$ Or FT their substitution $u=(2x+1)^{\frac{1}{2}}$ or $u^2=2x+1$ leading to an integrand of the form $A\left(2u^2-1\right)u^2$	3.1a	M1	
	Obtains correct lower limit for their substitution	1.1a	M1	
	Completes a reasoned argument to show the required result with $a=1$	2.1	R1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
18(b)	Integrates to obtain $\frac{1}{2}\frac{4u^{\frac{5}{2}}}{5}$ or $\frac{4u^{\frac{5}{2}}}{5}$ or $-\frac{1}{2}\frac{2u^{\frac{3}{2}}}{3}$ or $\frac{2u^{\frac{3}{2}}}{3}$	1.1a	M1	$\int_0^4 (4x+1)(2x+1)^{\frac{1}{2}}\,dx$ $=\frac{1}{2}\int_1^9 \left(2u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)du$ $=\frac{1}{2}\left[\frac{4u^{\frac{5}{2}}}{5} - \frac{2u^{\frac{3}{2}}}{3}\right]_1^9$ $=\frac{1}{2}\left[\left(\frac{4\times 9^{\frac{5}{2}}}{5} - \frac{2\times 9^{\frac{3}{2}}}{3}\right) - \left(\frac{4\times 1^{\frac{5}{2}}}{5} - \frac{2\times 1^{\frac{3}{2}}}{3}\right)\right]$ $=\frac{1322}{15}$
	Obtains $\frac{1}{2}\left(\frac{4u^{\frac{5}{2}}}{5} - \frac{2u^{\frac{3}{2}}}{3}\right)$ or $\frac{4u^{\frac{5}{2}}}{5} - \frac{2u^{\frac{3}{2}}}{3}$	1.1b	A1	
	Substitutes limits explicitly into their integrated expression of the form $Au^{\frac{5}{2}} - Bu^{\frac{3}{2}}$ Where A and B are both positive FT their non-zero a Condone omission of powers on substitution of 1	1.1a	M1	
	Completes argument to show the given result with no unrecovered slips.	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
18(c)	Explains that increasing the number of rectangles will lead to an increased value so the (improved) approximation will be greater than A	2.4	E1	Since the area of the rectangles is an underestimate, increasing their number will give an increased total $> A$ No matter how many rectangles are used their total will be less than the exact area so $< \frac{1322}{15}$
	Gives a reason why the approximation will be an under-estimate so will be less than $\frac{1322}{15}$ AG	2.4	E1	
	Subtotal		**2**	

	Question 18 Total		**11**	