| | | |
|---|---|---|
| **10** | $A(20,18,-12)\ B(12,-6,20)\ C(4,y,z)$ | |
| | $\overrightarrow{AC} = (4,y,z) - (20,18,-12) = (-16, y-18, z+12)$ | M1 |
| | $\overrightarrow{BC} = (4,y,z) - (12,-6,20) = (-8, y+6, z-20)$ | M1 |
| | $\left\|\overrightarrow{AC}\right\| = \sqrt{(-16)^2 + (y-18)^2 + (z+12)^2} = 24$ | M1 |
| | $y^2 - 36y + z^2 + 24z = -148$ (*) | M1 |
| | $\left\|\overrightarrow{BC}\right\| = \sqrt{(-8)^2 + (y+6)^2 + (z-20)^2} = 24$ | M1 |
| | $y^2 + 12y + z^2 - 40z = 76$ (**) | M1 |
| | (*) subtract (**):<br>$-48y + 64z = -224$<br>$4z + 14 = 3y$ | M1 |
| | $9(y^2+12y + z^2 - 40z) = 9(76)$<br>$9y^2 + 108y + 9z^2 - 360z = 684$ | M1 |
| | $(4z+14)^2 + 36(4z+14) + 9z^2 - 360z = 684$ | M1<br>Students may also eliminate $z$ |
| | $25z^2 - 104z + 16 = 0$<br>$(25z-4)(z-4) = 0$<br>$z = 4$ or $\frac{4}{25}$ | A1 |
| | $z = 4$:<br>$4(4) + 14 = 3y \Rightarrow y = 10$<br>$(4,10,4)$ | A1 |
| | $z = \frac{4}{25}$:<br>$4\left(\frac{4}{25}\right) + 14 = 3y \Rightarrow y = \frac{122}{25}$<br>$\left(4, \frac{122}{25}, \frac{4}{25}\right)$ | A1 |
| | | **[12]** |
| | | **[12 marks]** |