

7

A student notices that when he adds two consecutive odd numbers together the answer always seems to be the difference between two square numbers.

He claims that this will always be true.

He attempts to prove his claim as follows:

Step 1: Check first few cases

$$3 + 5 = 8 \text{ and } 8 = 3^2 - 1^2$$

$$5 + 7 = 12 \text{ and } 12 = 4^2 - 2^2$$

$$7 + 9 = 16 \text{ and } 16 = 5^2 - 3^2$$

Step 2: Use pattern to predict and check a large example

$$101 + 103 = 204$$

subtract 1 and divide by 2 for the first number

Add 1 and divide by two for the second number

$$52^2 - 50^2 = 204 \text{ it works!}$$

Step 3: Conclusion

The first few cases work and there is a pattern, which can be used to predict larger numbers.

Therefore, it must be true for all consecutive odd numbers.

7 (a) Explain what is wrong with the student's "proof".

[1 mark]

7 (b) Prove that the student's claim is correct.

[3 marks]