

**7**  $a$  and  $b$  are two positive irrational numbers.

The sum of  $a$  and  $b$  is rational.

The product of  $a$  and  $b$  is rational.

Caroline is trying to prove  $\frac{1}{a} + \frac{1}{b}$  is rational.

Here is her proof:

Step 1  $\frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$

Step 2 2 is rational and  $a+b$  is non-zero and rational.

Step 3 Therefore  $\frac{2}{a+b}$  is rational.

Step 4 Hence  $\frac{1}{a} + \frac{1}{b}$  is rational.

**7 (a) (i)** Identify Caroline's mistake.

**[1 mark]**

**7 (a) (ii)** Write down a correct version of the proof.

**[2 marks]**

**7 (b)** Prove by contradiction that the difference of any rational number and any irrational number is irrational.

**[4 marks]**