

**8** Kai is proving that  $n^3 - n$  is a multiple of 3 for all positive integer values of  $n$ .

Kai begins a proof by exhaustion.

Step 1  $n^3 - n = n(n^2 - 1)$

Step 2 When  $n = 3m$ , where  $m$  is a non-negative integer  $n^3 - n = 3m(9m^2 - 1)$   
which is a multiple of 3

Step 3 When  $n = 3m + 1$ ,  $n^3 - n = (3m + 1)((3m + 1)^2 - 1)$

Step 4  $= (3m + 1)(9m^2)$   
 $= 3(3m + 1)(3m^2)$   
which is a multiple of 3

Step 5 Therefore  $n^3 - n$  is a multiple of 3 for all positive integer values of  $n$

**8 (a)** Explain the two mistakes that Kai has made after Step 3.

**[2 marks]**

**8 (b)** Correct Kai's argument from Step 4 onwards.

**[4 marks]**