

9 A robotic arm which is attached to a flat surface at the origin O , is used to draw a graphic design.

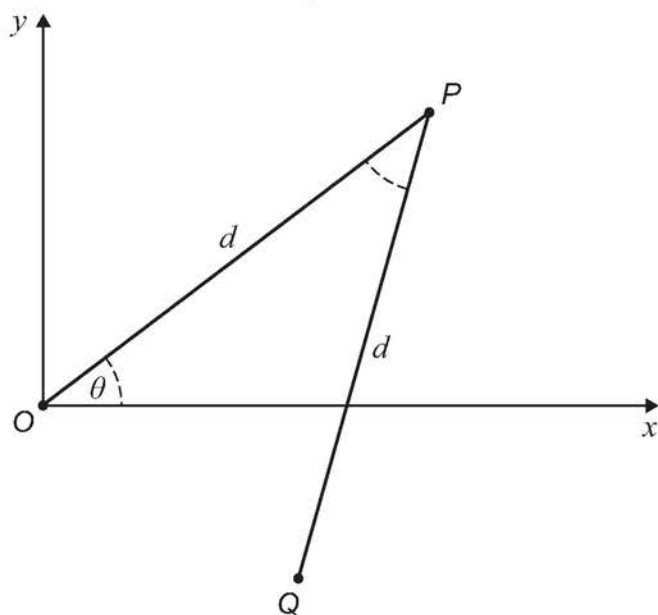
The arm is made from two rods OP and PQ , each of length d , which are joined at P .

A pen is attached to the arm at Q .

The coordinates of the pen are controlled by adjusting the angle OPQ and the angle θ between OP and the x -axis.

For this particular design the pen is made to move so that the two angles are always equal to each other with $0 \leq \theta \leq \frac{\pi}{2}$ as shown in **Figure 2**.

Figure 2



9 (a) Show that the x -coordinate of the pen can be modelled by the equation

$$x = d \left(\cos \theta + \sin \left(2\theta - \frac{\pi}{2} \right) \right)$$

[2 marks]

9 (b) Hence, show that

$$x = d(1 + \cos \theta - 2 \cos^2 \theta)$$

[2 marks]

9 (c) It can be shown that

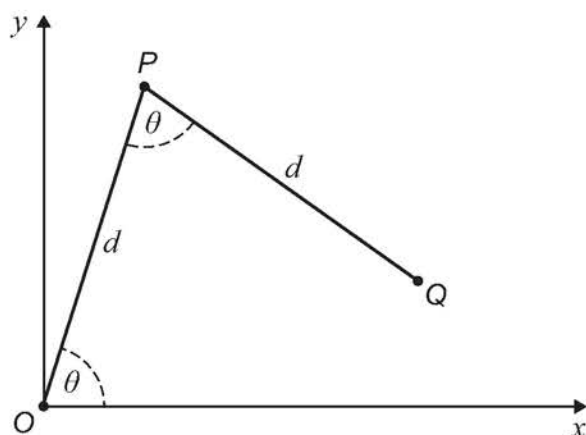
$$x = \frac{9d}{8} - d \left(\cos \theta - \frac{1}{4} \right)^2$$

State the greatest possible value of x and the corresponding value of $\cos \theta$

[2 marks]

9 (d) **Figure 3** below shows the arm when the x -coordinate is at its greatest possible value.

Figure 3



Find, in terms of d , the exact distance OQ .

[3 marks]