

10 A gardener has a greenhouse containing 900 tomato plants.

The gardener notices that some of the tomato plants are damaged by insects.

Initially there are 25 damaged tomato plants.

The number of tomato plants damaged by insects is increasing by 32% each day.

10 (a) The total number of plants damaged by insects, x , is modelled by

$$x = A \times B^t$$

where A and B are constants and t is the number of days after the gardener first noticed the damaged plants.

10 (a) (i) Use this model to find the total number of plants damaged by insects 5 days after the gardener noticed the damaged plants.

[3 marks]

10 (a) (ii) Explain why this model is not realistic in the long term.

[2 marks]

10 (b) A refined model assumes the rate of increase of the number of plants damaged by insects is given by

$$\frac{dx}{dt} = \frac{x(900 - x)}{2700}$$

10 (b) (i) Show that

$$\int \left(\frac{A}{x} + \frac{B}{900 - x} \right) dx = \int dt$$

where A and B are positive integers to be found.

[3 marks]

10 (b) (ii) Hence, find t in terms of x .

[5 marks]

10 (b) (iii) Hence, find the number of days it takes from when the damage is first noticed until half of the plants are damaged by the insects.

[2 marks]