Question	Scheme	Marks	AOs
6(a)	$n=1,\ \sum_{r=1}^{1} r^2 = 1$ and $\frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}(1)(2)(3) = 1$	B1	2.2a
	Assume general statement is true for $n=k$ So assume $\sum_{r=1}^{k} r^2 = \frac{1}{6}k(k+1)(2k+1)$ is true	M1	2.4
	$\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$	M1	2.1
	$= \frac{1}{6}(k+1)(2k^2+7k+6)$	A1	1.1b
	$= \frac{1}{6}(k+1)(k+2)(2k+3) = \frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$	A1	1.1b
	Then the general result is <u>true for $n=k+1$</u> As the general result has been shown to be <u>true for $n=1$,</u> then the general result <u>is true for all n</u> $\in \mathbb{Z}^+$	A1	2.4
		(6)	
(b)	$\sum_{r=1}^{n} r(r+6)(r-6) = \sum_{r=1}^{n} (r^3 - 36r)$		
	$= \frac{1}{4}n^2(n+1)^2 - \frac{36}{2}n(n+1)$	M1 A1	2.1 1.1b
	$= \frac{1}{4}n(n+1)\left[n(n+1)-72\right]$	M1	1.1b
	$= \frac{1}{4}n(n+1)(n-8)(n+9)$ * cso	A1*	1.1b
		(4)	
(c)	$\frac{1}{4}n(n+1)(n-8)(n+9) = \frac{17}{6}n(n+1)(2n+1)$	M1	1.1b
	$\frac{1}{4}(n-8)(n+9) = \frac{17}{6}(2n+1)$	M1	1.1b
	$3n^2 - 65n - 250 = 0$	A1	1.1b
	$(3n+10)(n-25) = 0$	M1	1.1b
	(As n must be a positive integer,) $n = 25$	A1	2.3
		(5)	
			(15 marks)

Question 6 notes:
(a) **B1:** Checks $n = 1$ works for both sides of the general statement **M1:** Assumes (general result) true for $n = k$ **M1:** Attempts to add $(k + 1)^{\text{th}}$ term to the sum of k terms **A1:** Correct algebraic work leading to **either** $\frac{1}{6}(k+1)(2k^2+7k+6)$ **or** $\frac{1}{6}(k+2)(2k^2+5k+3)$ **or** $\frac{1}{6}(2k+3)(k^2+3k+2)$ **A1:** Correct algebraic work leading to $\frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$ **A1:** cso leading to a correct induction statement conveying all three underlined points
(b) **M1:** Substitutes at least one of the standard formulae into their expanded expression **A1:** Correct expression **M1:** Depends on previous M mark. Attempt to factorise at least $n(n+1)$ having used **A1*:** Obtains $\frac{1}{4}n(n+1)(n-8)(n+9)$ by cso
(c) **M1:** Sets their part (a) answer equal to $\frac{17}{6}n(n+1)(2n+1)$ **M1:** Cancels out $n(n+1)$ from both sides of their equation **A1:** $3n^2 - 65n - 250 = 0$ **M1:** A valid method for solving a 3 term quadratic equation **A1:** Only one solution of $n = 25$