Question	Scheme	Marks	AOs
9(a)	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{ = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{ \overrightarrow{OF} = \mathbf{r} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\left\{ \overrightarrow{OF} \bullet \overrightarrow{AB} = 0 \Rightarrow \right\} \begin{pmatrix} -3+12\lambda \\ 1+3\lambda \\ -7+18\lambda \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = 0$ $\Rightarrow -36+144\lambda+3+9\lambda-126+324\lambda=0 \Rightarrow 477\lambda-159=0$	dM1	1.1b
	$\Rightarrow \lambda = \frac{1}{3}$	A1	1.1b
	$\left\{ \overrightarrow{OF} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and minimum distance $= \sqrt{(1)^2 + (2)^2 + (-1)^2}$	dM1	3.1a
	$= \sqrt{6}$ **or** 2.449…	A1	1.1b
	> 2, so the octopus is not able to catch the fish F	A1ft	3.2a
		(7)	

Question	Scheme	Marks	
	9(a) Alternative 1		
	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{ = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{ \overrightarrow{OA} = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \text{ and } \overrightarrow{AB} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \Rightarrow \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\cos\theta \left\{ = \dfrac{\overrightarrow{OA} \bullet \overrightarrow{AB}}{\left\lvert\overrightarrow{OA}\right\rvert . \left\lvert\overrightarrow{AB}\right\rvert} \right\} = \dfrac{\pm \left(\begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right)}{\sqrt{(-3)^2 + (1)^2 + (-7)^2} . \sqrt{(12)^2 + (3)^2 + (18)^2}}$	dM1	1.1b
	$\left\{ \cos\theta = \dfrac{-36 + 3 - 126}{\sqrt{59} . \sqrt{477}} = \dfrac{-159}{\sqrt{59} . \sqrt{477}} \right\}$		
	$\theta = 161.4038029...$ or $18.59619709...$ or $\sin\theta = 0.3188964021...$	A1	1.1b
	minimum distance $= \sqrt{(-3)^2 + (1)^2 + (-7)^2} \sin(18.59619709...)$	dM1	3.1a
	$= \sqrt{6}$ or $2.449\ldots$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F	A1ft	3.2a
		(7)	
	9(a) Alternative 2		
	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{ = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{ \overrightarrow{OF} = \mathbf{r} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\left\lvert\overrightarrow{OF}\right\rvert^2 = (-3 + 12\lambda)^2 + (1 + 3\lambda)^2 + (-7 + 18\lambda)^2$	dM1	1.1b
	$= 9 - 72\lambda + 144\lambda^2 + 1 + 6\lambda + 9\lambda^2 + 49 - 252\lambda + 324\lambda^2$		
	$= 477\lambda^2 - 318\lambda + 59$	A1	1.1b
	$= 53(3\lambda - 1)^2 + 6$	dM1	3.1a
	minimum distance $= \sqrt{6}$ or $2.449\ldots$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F	A1ft	3.2a
		(7)	

Question	Scheme	Marks	AOs
9(b)	e.g. Fish *F* may not swim in an exact straight line from *A* to *B* Fish *F* may hit an obstacle whilst swimming from *A* to *B* Fish *F* may deviate his path to avoid being caught by the octopus	B1	3.5b
		(1)	
(c)	e.g. Octopus is effectively modelled as a particle – so we may need to look at where the octopus's mass is distributed Octopus may during the fish *F*'s motion move away from its fixed location at *O*	B1	3.5b
		(1)	
			(9 marks)

Question 9 notes:

(a)

M1: Attempts to find $\overrightarrow{OB} - \overrightarrow{OA}$ **or** $\overrightarrow{OA} - \overrightarrow{OB}$ **or** the direction vector **d**

M1: Applies $\overrightarrow{OA} + \lambda$(their $\overrightarrow{AB}$ or their $\overrightarrow{BA}$ or their **d**) or equivalent

M1: Depends on previous M mark. Writes down
(their $\overrightarrow{OF}$ which is in terms of λ)•(their $\overrightarrow{AB}$) = 0. Can be implied

A1: Lambda is correct. e.g. $\lambda = \frac{1}{3}$ for $\overrightarrow{AB} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$ or $\lambda = 1$ for $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$

M1: Depends on previous M mark. Complete method for finding $\left|\overrightarrow{OF}\right|$

A1: $\sqrt{6}$ **or** awrt 2.4

A1ft : Correct follow through conclusion, which is in context with the question

Alternative 1

(a)

M1: Attempts to find $\overrightarrow{OB} - \overrightarrow{OA}$ **or** $\overrightarrow{OA} - \overrightarrow{OB}$ **or** the direction vector **d**

M1: Realisation that the dot product is required between $\overrightarrow{OA}$ and their $\overrightarrow{AB}$. (o.e.)

M1: Depends on previous M mark. Applies dot product formula between $\overrightarrow{OA}$ and their $\overrightarrow{AB}$ (o.e.)

A1: θ = awrt 161.4 or awrt 18.6 or $\sin\theta$ = awrt 0.319

M1: Depends on previous M mark. (their *OA*)sin(their θ)

A1: $\sqrt{6}$ or awrt 2.4

A1ft : Correct follow through conclusion, which is in context with the question

Question 9 notes continued:

Alternative 2

(a)

M1: Attempts to find $\overrightarrow{OB} - \overrightarrow{OA}$ **or** $\overrightarrow{OA} - \overrightarrow{OB}$ **or** the direction vector **d**

M1: Applies $\overrightarrow{OA} + \lambda$(their $\overrightarrow{AB}$ or their $\overrightarrow{BA}$ or their **d**) or equivalent

M1: Depends on previous M mark. Applies Pythagoras by finding $\left|\overrightarrow{OF}\right|^2$, o.e.

A1: $\left|\overrightarrow{OF}\right|^2 = 477\lambda^2 - 318\lambda + 59$

M1: Depends on previous M mark. Method of completing the square or differentiating their $\left|\overrightarrow{OF}\right|^2$ w.r.t. λ

A1: $\sqrt{6}$ or awrt 2.4

A1ft : Correct follow through conclusion, which is in context with the question

(b)

B1: An acceptable criticism for fish F, which is in context with the question

(c)

B1: An acceptable criticism for the octopus, which is in context with the question