

Question	Scheme	Marks	AOs
3(a)	$\begin{vmatrix} 3 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & k & 2 \end{vmatrix} = 3(3 \times 2 - k \times -1) - 2(2 \times 2 - 1 \times -1) + 1(2 \times k - 1 \times 3)$	M1	1.1b
	$= 5k + 5$	A1	1.1b
		(2)	
(b)	(i) $3x + 2y + z = 4$	B1	1.1b
	(ii) EITHER $y = 2 - \lambda \Rightarrow \lambda = 2 - y$		
	OR $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = 0$ and $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = 0 = 0 \Rightarrow \mathbf{n} = A \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$	M1	1.1b
	EITHER $x = 1 + (2 - y) + \mu \Rightarrow z = 3 - (2 - y) + 2(x - (2 - y) - 1)$		
	OR $d = A \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 5A$	M1	1.1b
	$\Rightarrow 2x + 3y - z = 5$	A1	1.1b
		(4)	
(c)	The planes meet when all three equations are satisfied, so we can find where they meet by solving $\begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & k & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix}$	B1	3.1a
	If the planes form a sheaf, then they must share a common line. But if $k \neq -1$ the determinant of the matrix is non-zero, so the equation has unique solution and hence the planes would meet in a single point. Therefore, we must have $k = -1$.	B1	2.3
		(4)	

(8 marks)

Notes:
<p>(a) M1: Attempts determinant. Correct $() - () + ()$ structure, but allow up to two slips in entries. A1: Determinant is $5k + 5$.</p>
<p>(b)(i) B1: Correct equation. (Accept multiples.)</p>
<p>(ii) M1: Eliminates λ or μ from Cartesian equations OR attempts to find a vector normal to both direction vectors using scalar product (or cross product may be used). M1: Eliminates the other variable from their equations OR uses the scalar product with $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and their normal to find d. A1: Correct equation. (Accept multiples.)</p>

(c)

B1: Makes the link between the Cartesian equations and the matrix in (a).

B1: Identifies a sheaf cannot be formed if the solution is unique and so the matrix must be singular to form a sheaf, and hence $k = -1$.