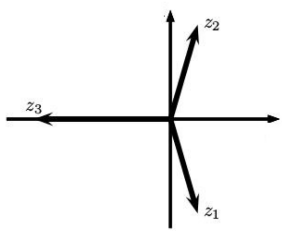
Question	Scheme	Marks	AOs
5	$\mathrm{f}(z) = 8z^3 + 12z^2 + 6z + 65$; root $\alpha = \frac{1}{2} - i\sqrt{3}$		
(a)	$\frac{1}{2} + \mathrm{i}\sqrt{3}$	B1	1.2
		(1)	
(b)	Attempts quadratic factor: $z^2 - z + \frac{13}{4}$ or $4z^2 - 4z + 13$	M1	1.1b
	So $\mathrm{f}(z) = (4z^2 - 4z + 13)(2z + 5)$ (oe)	M1	1.1b
	So roots are $z_1 = \frac{1}{2} - i\sqrt{3}$, $z_2 = \frac{1}{2} + i\sqrt{3}$ and $z_3 = -\frac{5}{2}$	A1	1.1b
		(3)	
(c)	 Correct complex roots	B1	1.1b
	Correct real root	B1ft	1.1b
		(2)	
(d)	E.g. $\lvert z_1 - z_2 \rvert = 2\sqrt{3}$, $\lvert z_1 - z_3 \rvert = \sqrt{\left(\frac{1}{2} + \frac{5}{2}\right)^2 + 3} = \sqrt{12} = 2\sqrt{3}$ and $\lvert z_2 - z_3 \rvert = 2\sqrt{3}$ by symmetry. OR $\arg(z_2 - z_3) = \arg\left(3 + \mathrm{i}\sqrt{3}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$, so (by symmetry) angle at z_3 is $2 \times \frac{\pi}{6} = \frac{\pi}{3}$, and since by symmetry the angles at z_1 and z_2 are equal, they must also each be $\frac{\pi}{3}$ (so all add to π).	M1	3.1a
	All three sides of the triangle are the same length, and so the vertices form an equilateral triangle. OR All three angles are $\frac{\pi}{3}$ and so the triangle formed by the vertices	A1	2.1

	is equilateral.		
		(2)	
(8 marks)			

Notes:

(a)

B1: Correct conjugate root.

(b)

M1: Attempts quadratic factor $z - 2\text{Re}(\alpha)z + |\alpha|^2$ or $(z-\alpha)(z-\alpha^*)$. As a minimum accept an attempt at the product of roots.

M1: Attempts to find the linear term, e.g. by factorisation or dividing by quadratic term or use of product of roots being 65.

A1: Correct solutions. All three must be stated. Ignore labelling. Answers only score zero marks in (b). Algebra must be used.

(c)

B1: Correct placement of complex roots, symmetric about real axis, in first and fourth quadrants, closer to imaginary axis than real axis. Lines/arrows not needed, just points.

B1ft: Correct placement for real root. If root is correct then on real axis further from origin than other roots, but follow through if a positive root found in (b).

(d)

M1: A complete method to find either all three sides or all three angles of the triangle.

A1: Sides/angles all correct from correct work/reasoning and conclusion made to draw the argument together.