Question	Scheme	Marks	AOs
6(a)	For $n=1$: $\mathrm{f}(1)=1^5+4(1)=5$. So the statement is true for $n=1$	B1	2.2a
	Assume true for $n=k..1$, so k^5+4k is divisible by 5	M1	2.4
	$\mathrm{f}(k+1)=(k+1)^5+4(k+1)=k^5+5k^4+\ldots$	M1	2.1
	$=k^5+5k^4+10k^3+10k^2+5k+1+4k+4$	A1	1.1b
	$=k^5+4k+5\left(k^4+2k^3+2k^2+k+1\right)$ $=\mathrm{f}(k)+5\left(k^4+2k^3+2k^2+k+1\right)$	A1	1.1b
	Hence the result is true for $n=k+1$. Since it is <u>true for $n=1$,</u> and <u>if true for $n=k$ then true for $n=k+1$</u>, thus by mathematical induction the <u>result holds for all n</u> $\in\square^+$	A1cso	2.4
		(6)	
(b)	For any x, $\mathrm{f}(-x)=\left((-x)^5+4(-x)=\right)-x^5-4x=-(x^5+4x)=-\mathrm{f}(x)$ *	B1*	2.5
		(1)	
(c)	We know from (a) that $\mathrm{f}(n)$ is divisible 5 for all positive integers, and since by (b) for negative n we have $\mathrm{f}(n)=-\mathrm{f}(-n)$ is divisible by 5 (as $-n$ is a positive integer).	M1	2.1
	Since $\mathrm{f}(0)=0$ is also divisible by 5, so $\mathrm{f}(n)$ is divisible by 5 for all integers n.	A1	2.4
		(2)	
			(9 marks)

Notes:

(a)

B1: Shows the statement is true for $n=1$.

M1: Makes the inductive assumption, assume true for $n=k$.

M1: Attempts to expand $\mathrm{f}(k+1)$ or $\mathrm{f}(k+1)-\mathrm{f}(k)$ using the binomial theorem.

A1: Correct expansion.

A1: Correct expression for $\mathrm{f}(k+1)$ with common factor 5 made clear.

A1: Completes the inductive argument conveying **all** three underlined points or equivalent at some point in their argument.

(b)

B1*: Correct proof with each of the un-bracketed expressions shown in the scheme.

(c)

M1: Reasons that result holds for negative integers by results of (a) and (b).

A1t: Considers the $n=0$ case and concludes true for all integers. Note: if $n=0$ case was used as the inductive base in (a), the reasoning here must clearly refer to all integers for the mark to be awarded.