Question	Scheme	Marks	AOs
7(a)	Explains that there are $0.23A_n$ new juveniles from births AND $\frac{2}{3}(0.87)J_n$ surviving juveniles staying juveniles	B1	2.4
	$J_{n+1} = 0.23A_n + \frac{2}{3}(0.87)J_n \; = 0.23A_n + 0.58J_n$	B1	1.1b
		(2)	
(b)	$p = 0.30$	B1	3.3
	$\mathbf{M} = \begin{pmatrix} 0.97 & 0.29 \\ 0.23 & 0.58 \end{pmatrix}$	B1ft B1	3.3 3.3
		(3)	
(c)(i)	$\begin{pmatrix} A_{-1} \\ J_{-1} \end{pmatrix} = \begin{pmatrix} 0.97 & 0.29 \\ 0.23 & 0.58 \end{pmatrix}^{-1} \begin{pmatrix} 1.2 \\ 0.3 \end{pmatrix} = \begin{pmatrix} 1.228\ldots \\ 0.0302\ldots \end{pmatrix}$ $\left(\text{NB } \mathbf{M}^{-1} = \begin{pmatrix} 1.1695\ldots & -0.58479\ldots \\ -0.46380\ldots & 1.9560\ldots \end{pmatrix} \right)$	M1	3.4
	Hence total population is $1.228\ldots + 0.0302\ldots = \ldots$	M1	1.1b
	$= 1.26$ million	A1	2.2b
		(3)	
(c)(ii)	$\begin{pmatrix} A_7 \\ J_7 \end{pmatrix} = \mathbf{M}^7 \begin{pmatrix} 1.2 \\ 0.3 \end{pmatrix} \left(= \begin{pmatrix} 2.117\ldots \\ 0.938\ldots \end{pmatrix} \right)$	M1	3.4
	So the juvenile population on 1st January 2025 is expected to be 0.938 million (or 938000)	A1	1.1b
		(2)	
(d)	For attempting to include 15000 juvenile being exported by adding (or subtracting) a suitable vector, ie. $\begin{pmatrix} A_{n+1} \\ J_{n+1} \end{pmatrix} = \mathbf{M} \begin{pmatrix} A_n \\ J_n \end{pmatrix} - \begin{pmatrix} 0 \\ \text{their value} \end{pmatrix}$	M1	3.5c
	$\begin{pmatrix} A_1 \\ J_1 \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.3 \end{pmatrix} \quad \begin{pmatrix} A_{n+1} \\ J_{n+1} \end{pmatrix} = \mathbf{M} \begin{pmatrix} A_n \\ J_n \end{pmatrix} - \begin{pmatrix} 0 \\ 0.015 \end{pmatrix} \quad n .. 1$	A1ft	3.3
		(2)	
(e)	E.g. The exportation may have an effect on the proportion of juveniles becoming adults/proportion who become adults each year may fluctuate/birth rates and death rates may change over time/two sub-populations may not sufficiently reflect the population.	B1	3.5b
		(1)	
			(13 marks)

Notes:

(a)

B1: Explains the two components of the equation with reference to surviving juveniles remaining juveniles and that new juveniles arise through birth.

B1: Correct equation.

(b)

B1: Correct value for p.

B1ft: One row or column correct, follow through their equation in (a) on second row.

B1: Completely correct matrix.

(c)(i)

M1: Uses the model to find the population for 2017, ie $n = -1$, using their $\mathbf{M}$ and initial vector. May see $\mathbf{M}^{-1}$ used, or the correct answer can imply the method

M1: Adds the two components of their vector to give the total population.

A1: awrt 1.26 million.

(c)(ii)

M1: Uses a calculator to evaluate $\mathbf{M}^7(1.2, 0.3)^{\mathrm{T}}$, or multiplies by $\mathbf{M}$ seven times oe. Awrt $(2.12, 0.938)^{\mathrm{T}}$, or just awrt 0.938 is sufficient for this mark.

A1: Concludes juvenile population is 0.938 million, or 938000.

(d)

M1: For incorporating the exportation into the model by subtracting (or adding if a negative entry is used) a vector with top entry zero and bottom entry an attempt at the 15 000 exported. So allow if an incorrect value of e.g. 15 000 or 0.15 is used as the value for this mark.

A1ft: Sets up the new system in full with **correct** vector subtracted (or added), but follow through on their p and $\mathbf{M}$. Allow if the range on n is omitted.

(e)

B1: Any valid limitation of the model - allow for limitations of the new model or original model. Some examples are given above but accept any sensible limitation.