Question	Scheme	Marks	AOs
9(a)	Forms a correct strategy to find the minimum distance between the comet and O. Using $\overrightarrow{OC}=146\mathbf{i}+234\mathbf{j}-85\mathbf{k}+\lambda(-21\mathbf{i}-33\mathbf{j}+13\mathbf{k})$ **Way 1** Attempts dot product of $\overrightarrow{OC}$ and the direction $\mathbf{d}$ to form an equation in λ, then uses λ to find min distance or its square. **Way 2** Attempts distance formula for $\overrightarrow{OC}$ in terms of λ, then completes the square in λ to find min distance or its square. **Way 3** Attempts dot product to find angle, θ between the line and $\overrightarrow{OX}=146\mathbf{i}+234\mathbf{j}-85\mathbf{k}$ and use a trigonometric approach to find the minimum distance within a right angle triangle.	M1	3.1a
	Way 1 $\overrightarrow{OC}\cdot\mathbf{d}=0\Rightarrow\begin{pmatrix}146-21\lambda\\234-33\lambda\\-85+13\lambda\end{pmatrix}\cdot\begin{pmatrix}-21\\-33\\13\end{pmatrix}=0\Rightarrow-3066+441\lambda-$ $7722+1089\lambda-1105+169\lambda=0\Rightarrow1699\lambda=11893\Rightarrow\lambda=\ldots$ **Way 2** Min distance, d, given by $d^2=(146-21\lambda)^2+(234-33\lambda)^2+(-85+113\lambda)^2=\ldots$ **Way 3** $\cos\theta=\left(\overrightarrow{OX}\cdot\mathbf{d}\right)/\left(\lvert\overrightarrow{OX}\rvert\lvert\mathbf{d}\rvert\right)$ $=\dfrac{\pm(146\times(-21)+234\times(-33)+(-85)\times13)}{\sqrt{146^2+234^2+(-85)^2}\sqrt{(-21)^2+(-33)^2+13^2}}=-0.9997\ldots$	M1	1.1b
	Way 1 $\lambda=7$ **Way 2** So $d^2=1699\lambda^2-23786\lambda+83297$ **Way 3** $\theta=178.653\ldots°$ or $1.34653\ldots°$ (oe) or $\sin\theta=0.023499\ldots$	A1	1.1b
	Way 1 So distance is $d=\sqrt{(146-7\times21)^2+(234-7\times33)^2+(-85+7\times13)^2}=\ldots\left(=\sqrt{46}=6.782\ldots\right)$ **Way 2** $=1699\left[(\lambda-7)^2-49\right]+83297=1699(\lambda-7)^2+46$, so $d_{\min}=\sqrt{\text{their } 46}$ or $d_{\min}^2=\text{their } 46$ **Way 3** So $d_{\min}=\sqrt{146^2+234^2+(-85)^2}\sin\theta=\ldots(=6.782\ldots)$	M1	1.1b
	Interprets situation correctly and compares their minimum distance with the radius of the planet with correct units, e.g. 6500km compared with 6782km or 6.5^2 with 46.	M1	3.1b
	The closest distance of the comet to the planet is more than a radius away from the centre, so comet (just) misses planet.	A1	3.2a
		(6)	

Question	Scheme	Marks	AOs
(b)	$C_{\lambda=4}=\begin{pmatrix}62\\102\\-33\end{pmatrix}$, so need $\mathbf{d}_1=\begin{pmatrix}5\\0\\12\end{pmatrix}-\begin{pmatrix}62\\102\\-33\end{pmatrix}$ and $\mathbf{d}_2=\begin{pmatrix}4\\12\\-3\end{pmatrix}-\begin{pmatrix}62\\102\\-33\end{pmatrix}$	M1	3.1a
	$\mathbf{d}_1=\begin{pmatrix}-57\\-102\\45\end{pmatrix}$ and $\mathbf{d}_2=\begin{pmatrix}-58\\-90\\30\end{pmatrix}$	A1	1.1b
	$\cos\angle ACB=\dfrac{(-57)(-58)+(-102)(-90)+(45)(30)}{\sqrt{(-57)^2+(-102)^2+45^2}\sqrt{(-58)^2+(-90)^2+30^2}}$ $\left(=\dfrac{13836}{\sqrt{15678}\sqrt{12364}}=0.9937\ldots\right)$	M1	1.1b
	$\angle ACB=6.4°$ (awrt) ($6.399\ldots°$)	A1	1.1b
		(4)	
(c)	The comet may not follow a straight line course, (as e.g. gravity when nearing the planet will affect it).	B1	3.2b
		(1)	
		(11 marks)	

Notes:

(a)

M1: Demonstration of a correct overall strategy to find the minimum distance, or its square, between the comet and O. See examples in scheme. There are other variations, e.g. via differentiating an expression for d^2 which follow a similar pattern.

M1: See scheme. A correct starting method, setting up an equation to find λ, or an expression for the square of distance, or correct equation to find an angle in an appropriate triangle.

A1: Correct λ (Way 1) or quadratic in λ (Way 2) or angle or its sine (Way 3). Equivalents in radians are $33.11809\ldots$ or $0.02350\ldots$.

M1: A correct attempt to find the minimum distance or the square of the minimum distance.

M1: Translates the information about the planet into the context of the question to draw an appropriate comparison between the minimum distance, d, between comet and O with the radius, or compares d^2 with the square of the radius. Units must match (ie both in km or in thousands of km).

A1: Correct conclusion drawn, following a correct minimum distance found.

(b)

M1: Identifies the correct direction vectors required to find the angle.

A1: Correct two vectors found. Any (non-zero) multiples of these are fine.

M1: Applies dot product formula with their direction vectors.

A1: Correct angle.

(c)

B1: See scheme. Accept any other reasonable comment, e.g. comet may lose mass as it travels, and this may affect its motion, satellites may not be in exactly the same positions and so on.