

3. (a) Find, in terms of the real constant k , the determinant of the matrix

$$\mathbf{M} = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & k & 2 \end{pmatrix}$$

(2)

Three distinct planes, Π_1 , Π_2 and Π_3 , are defined by the equations

$$\Pi_1 : \mathbf{r} \cdot \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = 4$$

$$\Pi_2 : \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$\Pi_3 : x + ky + 2z = -1$$

where λ and μ are scalar parameters.

(b) Find an equation in Cartesian form for

(i) Π_1

(ii) Π_2

(4)

Given that the three planes Π_1 , Π_2 and Π_3 form a sheaf,

(c) use the answer to part (a) to explain why $k = -1$

(2)