7. The population of Zebu cattle in a particular country is modelled by two sub-populations, adults and juveniles. In this model, the only factors affecting the population of the Zebu are the birth and survival rates of the population.

Data recorded in the years preceding 2018 was used to suggest the annual birth and survival rates of the population.
The results are shown in the table below, with values to 2 significant figures. It is assumed that these rates will remain the same in future years.

	Birth rate	Survival rate
Adult population	0.23	0.97
Juvenile population	0	0.87

It is also assumed that $\frac{1}{3}$ of the surviving juvenile population become adults each year.

Let A_n and J_n be the respective sub-populations, in millions, of adults and juveniles, n years after 1st January 2018. Then the adult population in year $n + 1$ satisfies the equation

$$A_{n+1} = 0.97A_n + \frac{1}{3}(0.87)J_n = 0.97A_n + 0.29J_n$$

(a) Form the corresponding equation for the juvenile population in year $n + 1$ under this model, justifying your values.

(2)

The total population on 1st January 2018 was estimated, to 2 significant figures, as 1.5 million Zebu, with 1.2 million of these being adults.

(b) Find the value of p and the matrix $\mathbf{M}$ such that the population of Zebu can be modelled by the system

$$\begin{pmatrix} A_0 \\ J_0 \end{pmatrix} = \begin{pmatrix} 1.2 \\ p \end{pmatrix} \qquad \begin{pmatrix} A_{n+1} \\ J_{n+1} \end{pmatrix} = \mathbf{M}\begin{pmatrix} A_n \\ J_n \end{pmatrix}$$

Give p to 2 significant figures and each entry of $\mathbf{M}$ to 2 decimal places.

(3)

Using the model formed in (b), find, to 3 significant figures,

(c) (i) the **total** Zebu population that was present on 1st January 2017

(ii) the predicted **juvenile** Zebu population on 1st January 2025

(5)

As a result of the predictions of this model, it is decided that the country will export 15 000 juveniles to a neighbouring country at the end of each year.

(d) Adapt the model from 2018 onwards to include this export.

(2)

(e) State one limitation of this model.

(1)