| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4(a)** | Attempts the scalar product between the direction of W and the normal to the road and uses trigonometry to find an angle. | M1 | 3.1a |
| | $\left(\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}-1\\-1\\-3\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=-9$ or $\left(\begin{pmatrix}-1\\-1\\-3\end{pmatrix}-\begin{pmatrix}1\\2\\-3\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=9$ | M1<br>A1 | 1.1b<br>1.1b |
| | $\sqrt{(2)^2+(3)^3+(0)^2}\sqrt{(3)^2+(-5)^3+(-18)^2}\cos\alpha="-9"$<br>$\theta=90-\arccos\left(\frac{9}{\sqrt{13}\sqrt{358}}\right)$ or $\theta=\arcsin\left(\frac{9}{\sqrt{13}\sqrt{358}}\right)$<br>Angle between pipe and road $=7.58^\circ$ (3sf) or 0.132 radians (3sf)<br>(Allow $-7.58^\circ$ or $-0.132$ radians) | M1<br>A1 | 1.1b<br>3.2a |
| | | **(5)** | |
| **(b)** | $W:\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}$ or $\begin{pmatrix}1\\2\\-3\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\0\end{pmatrix}$ | B1ft | 1.1b |
| | $C\text{ to }W:\left\{\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right\}$ or $\left\{\begin{pmatrix}1\\2\\-3\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right\}$ | M1 | 3.4 |
| | $\begin{pmatrix}2t\\3t+1\\-3\end{pmatrix}\bullet\begin{pmatrix}2\\3\\0\end{pmatrix}=0\Rightarrow t=\ldots$ or $\begin{pmatrix}2+2\lambda\\4+3\lambda\\-3\end{pmatrix}\bullet\begin{pmatrix}2\\3\\0\end{pmatrix}=0\Rightarrow\lambda=\ldots$<br>or<br>$(2t)^2+(3t+1)^2+(-3)^2=\ldots$ or $(2+2t)^2+(4+3t)^2+(-3)^2=\ldots$ | M1 | 3.1b |
| | $t=-\frac{3}{13}$ or $\lambda=-\frac{16}{13}\Rightarrow(C\text{ to }W)_{\min}$ is $-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$<br>or<br>$(2t)^2+(3t+1)^2+(-3)^2=13\left(t+\frac{3}{13}\right)^2+\frac{121}{13}$<br>or<br>$(2+2t)^2+(4+3t)^2+(-3)^2=13\left(\lambda+\frac{16}{13}\right)^2+\frac{121}{13}$<br>or<br>$\frac{\mathrm{d}\left((2t)^2+(3t+1)^2+(-3)^2\right)}{\mathrm{d}t}=0\Rightarrow t=-\frac{3}{13}\Rightarrow C\text{ to }W$ is $-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$<br>Or<br>$\frac{\mathrm{d}\left((2+2t)^2+(4+3t)^2+(-3)^2\right)}{\mathrm{d}t}=0\Rightarrow t=-\frac{16}{13}\Rightarrow(C\text{ to }W)_{\min}$ is $-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$ | A1 | 1.1b |
| | $d=\sqrt{\left(-\frac{6}{13}\right)^2+\left(\frac{4}{13}\right)^2+(-3)^2}$ or $d=\sqrt{\frac{121}{13}}$ | **dd**M1 | 1.1b |

| | | | |
|---|---|---|---|
| | Shortest length of pipe needed is 305 or 305 cm or 3.05 m | A1 | 3.2a |
| | | **(6)** | |
| | | | **(11 marks)** |

**Notes**

(a)

M1: Realises the scalar product between the direction of *W* and the normal to the road is needed and so applies it and uses trigonometry to find an angle

M1: Calculates the scalar product between $\pm\left(\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}-1\\-1\\-3\end{pmatrix}\right)$ and $\pm\begin{pmatrix}3\\-5\\-18\end{pmatrix}$ (Allow sign slips as long as the intention is clear)

A1: $\begin{pmatrix}2\\3\\0\end{pmatrix}\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=-9$ or $\begin{pmatrix}-2\\-3\\0\end{pmatrix}\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=9$ or $\begin{pmatrix}2\\3\\0\end{pmatrix}\bullet\begin{pmatrix}-3\\5\\18\end{pmatrix}=9$ or $\begin{pmatrix}-2\\-3\\0\end{pmatrix}\bullet\begin{pmatrix}-3\\5\\18\end{pmatrix}=-9$

M1: A fully complete and correct method for obtaining the acute angle

A1: Awrt 7.58$^{\circ}$ or awrt 0.132 radians **(must see units)**. Do not isw and withhold this mark if extra answers are given.

(b)

B1ft: Forms the correct parametric form for the pipe *W*. Follow through their direction vector for *W* from part (a).

M1: Identifies the need to and forms a vector connecting *C* to *W* using a parametric form for *W*

M1: Uses the model to form the scalar product of *C* to *W* and the direction of *W* to find the value of their parameter **or** finds the distance *C* to *W* or ($C$ to $W$)$^2$ in terms of their parameter

A1: Correct vector or correct completion of the square

ddM1: Correct use of Pythagoras on their vector *CW* or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.**

A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m

**<u>Alternatives for part (b):</u>**

| | | | |
|---|---|---|---|
| **4(b) Way 2** | $\mathbf{AC}=\begin{pmatrix}0\\1\\-3\end{pmatrix},\quad \mathbf{AB}=\begin{pmatrix}2\\3\\0\end{pmatrix}$ | B1ft | 1.1b |
| | $\mathbf{AC.AB}=\begin{pmatrix}0\\1\\-3\end{pmatrix}.\begin{pmatrix}2\\3\\0\end{pmatrix}=3$ | M1 | 3.4 |
| | $\Rightarrow \cos CAB=\dfrac{3}{\sqrt{10}\sqrt{13}}\Rightarrow CAB=...$ | M1 | 3.1b |
| | $CAB=74.74....^{\circ}$ | A1 | 1.1b |
| | $d=\sqrt{10}\sin 74.74....^{\circ}$ | **dd**M1 | 1.1b |
| | Shortest length of pipe needed is 305 or 305 cm or 3.05 m | A1 | 3.2a |
| | | **(6)** | |

| | Notes | | |
|---|---|---|---|
| | (b)<br>B1ft: Forms the correct vectors. Follow through their direction vector for *W* from part (a).<br>M1: Identifies the need to and forms the scalar product between **AC** and **AB**<br>M1: Uses the model to form the scalar product and uses this to find the angle *CAB*<br>A1: Correct angle<br>ddM1: Correct method using their values or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.**<br>A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m | | |

| | | | |
|---|---|---|---|
| **4(b) Way 3** | $\mathbf{AC} = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad \mathbf{AB} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$ | B1ft | 1.1b |
| | $\mathbf{AC} \times \mathbf{AB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 3 \\ 2 & 3 & 0 \end{vmatrix} = \begin{pmatrix} -9 \\ 6 \\ 2 \end{pmatrix}$ | M1 | 3.4 |
| | $\lvert \mathbf{AC} \times \mathbf{AB} \rvert = \sqrt{9^2 + 6^2 + 2^2} = \ldots$ | M1 | 3.1b |
| | $= 11$ | A1 | 1.1b |
| | $d = \dfrac{11}{\lvert \mathbf{AB} \rvert} = \dfrac{11}{\sqrt{2^2 + 3^2}} = \ldots$ | **dd**M1 | 1.1b |
| | Shortest length of pipe needed is 305 or 305 cm or 3.05 m | A1 | 3.2a |
| | | **(6)** | |
| | Notes | | |
| | (b)<br>B1ft: Forms the correct vectors. Follow through their direction vector for *W* from part (a).<br>M1: Identifies the need to and forms the vector product between **AC** and **AB**<br>M1: Uses the model to find the magnitude of their vector product<br>A1: Correct value<br>ddM1: Correct method using their values or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.**<br>A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m | | |