

Question	Scheme	Marks	AOs
7	Complex roots are e.g. $\alpha \pm \beta i$ or $(z^3 + z^2 + pz + q) \div (z - 3) = z^2 + 4z + p + 12$ or $f(3) = 0 \Rightarrow 3^3 + 3^2 + 3p + q = 0$ or One of: $3 + z_2 + z_3 = -1$ , $3z_2z_3 = -q$ , $3z_2 + 3z_3 + z_2z_3 = p$	B1	3.1a
	Sum of roots $\alpha + \beta i + \alpha - \beta i + 3 = -1 \Rightarrow \alpha = \dots$ or $\alpha + \beta i + \alpha - \beta i = -4 \Rightarrow \alpha = \dots$	M1	1.1b
	$\alpha = -2$	A1	1.1b
	So $\frac{1}{2} \times 2\beta \times 5 = 35 \Rightarrow \beta = 7$	M1	1.1b
	$q = -3(-2 + 7i)(-2 - 7i) = \dots$ or $p = 3(-2 + 7i) + 3(-2 - 7i) + (-2 + 7i)(-2 - 7i)$ or $(z - 3)(z - (-2 + 7i))(z - (-2 - 7i)) = \dots$	M1	3.1a
	$q = -159$ or $p = 41$	A1	1.1b
	$3p + q = -36 \Rightarrow p = \frac{-36 - q}{3} = 41$ and $q = -159$	A1	1.1b
	(7)		
	<b>Alternative</b>		
	$(z^3 + z^2 + pz + q) \div (z - 3) = z^2 + 4z + p + 12$	B1	3.1a
	$z^2 + 4z + p + 12 = 0 \Rightarrow z = \frac{-4 \pm \sqrt{4^2 - 4(p + 12)}}{2} (= -2 \pm i\sqrt{p + 8})$	M1	1.1b
	$\alpha = -2$	A1	1.1b
	$\beta = \sqrt{p + 8}$	M1	1.1b
	$\frac{1}{2} \times (3 + 2) \times 2\sqrt{p + 8} = 35 \Rightarrow p = \dots$	M1	3.1a
	$p = 41$	A1	1.1b
$3p + q = -36 \Rightarrow q = -159$	A1	1.1b	
	(7)		

(7 marks)

## Notes

**B1:** Recognises that the other roots must form a conjugate pair **or** obtains  $z^2 + 4z + p + 12$  (or  $z^2 + 4z - \frac{q}{3}$ ) as the quadratic factor **or** writes down a correct equation for  $p$  and  $q$  **or** writes down a correct equation involving " $z_2$ " and " $z_3$ "

**M1:** Uses the sum of the roots of the cubic or the sum of the roots of their quadratic to find a value for " $\alpha$ "

**A1:** Correct value for " $\alpha$ "

**M1:** Uses their value for " $\alpha$ " and the given area to find a value for " $\beta$ ". Must be using the area and triangle dimensions correctly e.g.  $\frac{1}{2} \times \beta \times 5 = 35 \Rightarrow \beta = 14$  scores M0

**M1:** Uses an appropriate method to find  $p$  or  $q$

**A1:** A correct value for  $p$  or  $q$

**A1:** Correct values for  $p$  and  $q$

## Alternative

**B1:** Obtains  $z^2 + 4z + p + 12$  (or  $z^2 + 4z - \frac{q}{3}$ ) as the quadratic factor

**M1:** Solves their quadratic factor by completing the square or using the quadratic formula

**A1:** Correct value for " $\alpha$ "

**M1:** Uses their imaginary part to find " $\beta$ " in terms of  $p$

**M1:** Draws together the fact that the imaginary parts of their complex conjugate pair and the real root form the sides of the required triangle and forms an equation in terms of  $p$ , sets equal to 35 and solves for  $p$

**A1:** A correct value for  $p$  or  $q$

**A1:** Correct values for  $p$  and  $q$