Question	Scheme	Marks	AOs
8(i)	$n=1, \begin{pmatrix}5 & -8\\2 & -3\end{pmatrix}^1 = \begin{pmatrix}5 & -8\\2 & -3\end{pmatrix}, \quad \begin{pmatrix}4\times1+1 & -8(1)\\2\times1 & 1-4(1)\end{pmatrix} = \begin{pmatrix}5 & -8\\2 & -3\end{pmatrix}$ So the result is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $\begin{pmatrix}5 & -8\\2 & -3\end{pmatrix}^k = \begin{pmatrix}4k+1 & -8k\\2k & 1-4k\end{pmatrix}$	M1	2.4
	$\begin{pmatrix}5 & -8\\2 & -3\end{pmatrix}^{k+1} = \begin{pmatrix}4k+1 & -8k\\2k & 1-4k\end{pmatrix}\begin{pmatrix}5 & -8\\2 & -3\end{pmatrix}$ or $\begin{pmatrix}5 & -8\\2 & -3\end{pmatrix}^{k+1} = \begin{pmatrix}5 & -8\\2 & -3\end{pmatrix}\begin{pmatrix}4k+1 & -8k\\2k & 1-4k\end{pmatrix}$	M1	1.1b
	$\begin{pmatrix}4k+1 & -8k\\2k & 1-4k\end{pmatrix}\begin{pmatrix}5 & -8\\2 & -3\end{pmatrix} = \begin{pmatrix}5(4k+1)-16k & -8(4k+1)+24k\\10k+2(1-4k) & -16k-3(1-4k)\end{pmatrix}$ or $\begin{pmatrix}5 & -8\\2 & -3\end{pmatrix}\begin{pmatrix}4k+1 & -8k\\2k & 1-4k\end{pmatrix} = \begin{pmatrix}5(4k+1)-16k & -40k-8(1-4k)\\2(1+4k)-6k & -16k-3(1-4k)\end{pmatrix}$	A1	1.1b
	$= \begin{pmatrix}4(k+1)+1 & -8(k+1)\\2(k+1) & 1-4(k+1)\end{pmatrix}$	A1	2.1
	<u>If true for $n = k$</u> then <u>true for $n = k + 1$</u>, <u>true for $n = 1$</u> so <u>true for all (positive integers) n</u> (Allow "for all values")	A1	2.4
		(6)	
(ii) **Way 1**	f(k + 1) – f(k)		
	When $n = 1,\ 4^{n+1} + 5^{2n-1} = 16 + 5 = 21$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $4^{k+1} + 5^{2k-1}$ is divisible by 21	M1	2.4
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 4^{k+2} + 5^{2k+1} - 4^{k+1} - 5^{2k-1}$	M1	2.1
	$= 4\times4^{k+1} + 25\times5^{2k-1} - 4^{k+1} - 5^{2k-1}$		
	$= 3\mathrm{f}(k) + 21\times5^{2k-1}$ or e.g. $= 24\mathrm{f}(k) - 21\times4^{k+1}$	A1	1.1b
	$\mathrm{f}(k+1) = 4\mathrm{f}(k) + 21\times5^{2k-1}$ or e.g. $\mathrm{f}(k+1) = 25\mathrm{f}(k) - 21\times4^{k+1}$	A1	1.1b
	<u>If true for $n = k$</u> then <u>true for $n = k + 1$</u>, <u>true for $n = 1$</u> so <u>true for all (positive integers) n</u> (Allow "for all values")	A1	2.4
		(6)	

(ii) Way 2	f(*k* + 1)		
	When $n = 1$, $4^{n+1} + 5^{2n-1} = 16 + 5 = 21$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $4^{k+1} + 5^{2k-1}$ is divisible by 21	M1	2.4
	$\mathrm{f}\left(k+1\right) = 4^{k+1+1} + 5^{2(k+1)-1}$	M1	2.1
	$\mathrm{f}\left(k+1\right) = 4\times4^{k+1} + 5^{2k+1} = 4\times4^{k+1} + 4\times5^{2k-1} + 25\times5^{2k-1} - 4\times5^{2k-1}$ $\mathrm{f}\left(k+1\right) = 4\mathrm{f}\left(k\right) + 21\times5^{2k-1}$	A1 A1	1.1b 1.1b
	<u>If true for *n* = *k*</u> then <u>true for *n* = *k* + 1</u>, <u>true for *n* = 1</u> so <u>true for all (positive integers) *n*</u> (Allow "for all values")	A1	2.4
		(6)	
(ii) Way 3	f(*k* + 1) – *m*f(*k*)		
	When $n = 1$, $4^{n+1} + 5^{2n-1} = 16 + 5 = 21$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $4^{k+1} + 5^{2k-1}$ is divisible by 21	M1	2.4
	$\mathrm{f}\left(k+1\right) - m\mathrm{f}\left(k\right) = 4^{k+2} + 5^{2k+1} - m\left(4^{k+1} + 5^{2k-1}\right)$	M1	2.1
	$= \left(4-m\right)4^{k+1} + 5^{2k+1} - m\times5^{2k-1}$ $= \left(4-m\right)\left(4^{k+1} + 5^{2k-1}\right) + 21\times5^{2k-1}$	A1	1.1b
	$= \left(4-m\right)\left(4^{k+1} + 5^{2k-1}\right) + 21\times5^{2k-1} + m\mathrm{f}\left(k\right)$	A1	1.1b
	<u>If true for *n* = *k*</u> then <u>true for *n* = *k* + 1</u>, <u>true for *n* = 1</u> so <u>true for all (positive integers) *n*</u> (Allow "for all values")	A1	2.4
		(6)	
(ii) Way 4	f(*k*) = 21*M*		
	When $n = 1$, $4^{n+1} + 5^{2n-1} = 16 + 5 = 21$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $4^{k+1} + 5^{2k-1} = 21M$	M1	2.4
	$\mathrm{f}\left(k+1\right) = 4^{k+1+1} + 5^{2(k+1)-1}$	M1	2.1
	$\mathrm{f}\left(k+1\right) = 4\times4^{k+1} + 5^{2k+1} = 4\left(21M - 5^{2k-1}\right) + 5^{2k+1}$ $\mathrm{f}\left(k+1\right) = 84M + 21\times5^{2k-1}$	A1 A1	1.1b 1.1b
	<u>If true for *n* = *k*</u> then <u>true for *n* = *k* + 1</u>, <u>true for *n* = 1</u> so <u>true for all (positive integers) *n*</u> (Allow "for all values")	A1	2.4
		(6)	

(12 marks)

Notes
(i) B1: Shows that the result holds for $n = 1$. Must see **substitution** into the rhs. The minimum would be: $\begin{pmatrix} 4+1 & -8 \\ 2 & 1-4 \end{pmatrix} = \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$. M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.) M1: Sets up a correct multiplication statement either way round A1: Achieves a correct un-simplified matrix A1: Reaches a correct simplified matrix with no errors **and the correct un-simplified matrix seen previously.** Note that the simplified result may be proved by equivalence. A1: Correct conclusion. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. (ii) **Way 1** B1: Shows that f(1) = 21 M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.) M1: Attempts $f(k + 1) – f(k)$ or equivalent work A1: Achieves a correct expression for $f(k + 1) – f(k)$ in terms of $f(k)$ A1: Reaches a correct expression for $f(k + 1)$ in terms of $f(k)$ A1: Correct conclusion. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. **Way 2** B1: Shows that f(1) = 21 M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.) M1: Attempts $f(k + 1)$ A1: Correctly obtains $4f(k)$ **or** $21 \times 5^{2k-1}$ A1: Reaches a correct expression for $f(k + 1)$ in terms of $f(k)$ A1: Correct conclusion. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. **Way 3** B1: Shows that f(1) = 21 M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.) M1: Attempts $f(k + 1) – mf(k)$ A1: Achieves a correct expression for $f(k + 1) – mf(k)$ in terms of $f(k)$ A1: Reaches a correct expression for $f(k + 1)$ in terms of $f(k)$ A1: Correct conclusion. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution.

Way 4

B1: Shows that f(1) = 21

M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.)

M1: Attempts f(k + 1)

A1: Correctly obtains $84M$ **or** $21\times5^{2k-1}$

A1: Reaches a correct expression for f(k + 1) in terms of M and 5^{2k-1}

A1: Correct conclusion. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution.