

8. (i) Prove by induction that for  $n \in \mathbb{Z}^+$

$$\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^n = \begin{pmatrix} 4n + 1 & -8n \\ 2n & 1 - 4n \end{pmatrix} \tag{6}$$

(ii) Prove by induction that for  $n \in \mathbb{Z}^+$

$$f(n) = 4^{n+1} + 5^{2n-1}$$

is divisible by 21

(6)