

9.

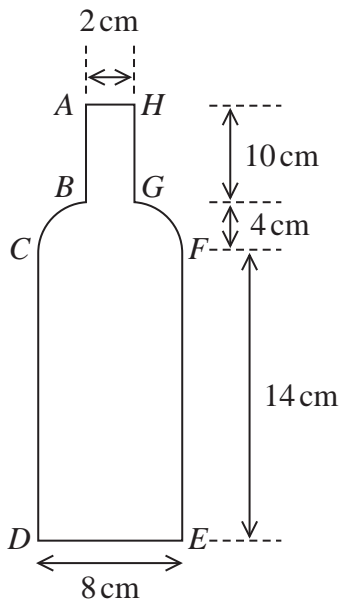


Figure 1

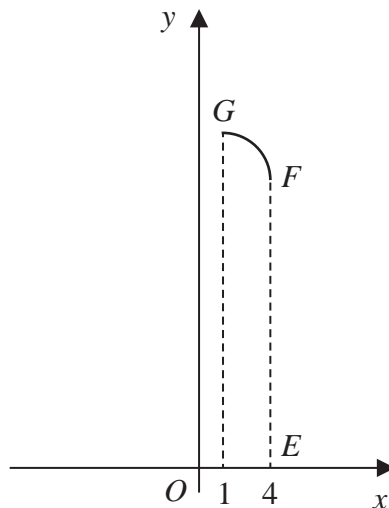


Figure 2

A mathematics student is modelling the profile of a glass bottle of water. Figure 1 shows a sketch of a central vertical cross-section  $ABCDEFGH$  of the bottle with the measurements taken by the student.

The horizontal cross-section between  $CF$  and  $DE$  is a circle of diameter 8 cm and the horizontal cross-section between  $BG$  and  $AH$  is a circle of diameter 2 cm.

The student thinks that the curve  $GF$  could be modelled as a curve with equation

$$y = ax^2 + b \quad 1 \leq x \leq 4$$

where  $a$  and  $b$  are constants and  $O$  is the fixed origin, as shown in Figure 2.

(a) Find the value of  $a$  and the value of  $b$  according to the model.

(2)

(b) Use the model to find the volume of water that the bottle can contain.

(7)

(c) State a limitation of the model.

(1)

The label on the bottle states that the bottle holds approximately  $750 \text{ cm}^3$  of water.

(d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning.

(1)