Question	Scheme	Marks	AOs
1. (a)	$(\det(\mathbf{M}) =)\ (4)(-7)-(2)(-5)$	M1	1.1a
	$\mathbf{M}$ is non-singular because $\det(\mathbf{M}) = -18$ and so $\det(\mathbf{M}) \neq 0$	A1	2.4
		(2)	
(b)	$\text{Area } R = \dfrac{\text{Area } S}{(\pm)\lvert\det \mathbf{M}\rvert} = \ldots$	M1	1.2
	$\text{Area}(R) = \dfrac{63}{\lvert -18\rvert} = \dfrac{7}{2}$ oe	A1ft	1.1b
		(2)	
(c)	$\begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}\begin{pmatrix} x \\ 2x \end{pmatrix} = \begin{pmatrix} 4x-10x \\ 2x-14x \end{pmatrix}$	M1	1.1b
	$= \begin{pmatrix} -6x \\ -12x \end{pmatrix}$ and so all points on $y = 2x$ map to points on $y = 2x$, hence the line is invariant. OR $= -6\begin{pmatrix} x \\ 2x \end{pmatrix}$ hence $y = 2x$ is invariant.	A1	2.1
		(2)	
		(6 marks)	

Notes

(a)	**M1**	An attempt to find $\det(\mathbf{M})$. Just the calculation is sufficient. Site of −18 implies this mark, which may be embedded in an attempt at the inverse..
	A1	$\det(\mathbf{M}) = -18$ **and** reference to zero, e.g. $-18 \neq 0$ **and** conclusion. The conclusion may precede finding the determinant (e.g. "Non-singular if $\det(\mathbf{M}) \neq 0$, $\det(\mathbf{M}) = -18 \neq 0$" is sufficient or accept "Non-singular if $\det(\mathbf{M}) \neq 0$, $\det(\mathbf{M}) = -18$, therefore non-singular" or some other indication of conclusion.) Need not mention "det(**M**)" to gain both marks here, a correct calculation, statement $-18\neq 0$, and conclusion hence **M** is non-singular can gain M1A1.
(b)	**M1**	Recalls determinant is needed for area scale factor by dividing 63 by ±their determinant.
	A1ft	$\dfrac{7}{2}$ or follow through $\dfrac{63}{\lvert \text{their det} \rvert}$. Must be positive and should be simplified to single fraction or exact decimal. (Allow if made positive following division by a negative determinant.)
(c)	**M1**	Attempts the matrix multiplication shown or with equivalent, e.g $\begin{pmatrix} \frac{1}{2}y \\ y \end{pmatrix}$. May use $\begin{pmatrix} x \\ y \end{pmatrix}$ **and** substitute $y = 2x$ later and this is fine for the method.
	A1	Correct multiplication and working leading to conclusion that the line is invariant. If the −6 is not extracted, they must make reference to image points being on line $y = 2x$. If the −6 is extracted to show it is a multiple of $\begin{pmatrix} x \\ 2x \end{pmatrix}$ followed by a conclusion "invariant" as minimum.

Notes Continued			
Alt for (c)	$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{-18}\begin{pmatrix} -7 & 5 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} x \\ 2x \end{pmatrix} = \frac{-1}{18}\begin{pmatrix} -7x+10x \\ -2x+8x \end{pmatrix}$	**M1**	1.1b
	$= \frac{-1}{18}\begin{pmatrix} 3x \\ 6x \end{pmatrix}\left(= \frac{-1}{6}\begin{pmatrix} x \\ 2x \end{pmatrix}\right) \Rightarrow b = 2a$ so points on line $y = 2x$ map to points on $y= 2x$, hence it is invariant.	**A1**	2.1
	Marks as per main scheme,		
Alt 2	(Since linear transformations map straight lines to straight lines…) E.g. $(1,2)$ is on line $y = 2x$, and $\begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4-10 \\ 2-14 \end{pmatrix}$	**M1**	1.1b
	$= \begin{pmatrix} -6 \\ -12 \end{pmatrix}$, which is also on the line $y=2x$, hence as (0,0) and (1,2) both map to points on $y = 2x$ (and transformation is linear) then $y =2x$ is invariant.	**A1**	2.1
	Notes		
	M1 — Identifies a point on the line $y = 2x$ and finds its image under T. If (0,0) is used there must be a clear statement it is because this is on the line, but for other points accept with any line on $y = 2x$ without statement.		
	A1 — Shows the image and another point, which may be (0,0), on $y=2x$ both map to points on $y = 2x$ concludes line is invariant. Need not reference transformation being linear for either mark here.		
Alt 3	$\begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}\begin{pmatrix} x \\ mx+c \end{pmatrix} = \begin{pmatrix} X \\ mX+c \end{pmatrix} \Rightarrow \begin{array}{c} 4x-5(mx+c) = X \\ 2x-7(mx+c) = mX+c \end{array}$ $\Rightarrow 2x-7(mx+c) = m\left(4x-5(mx+c)\right)+c$ $\Rightarrow \left(5m^2-11m+2\right)x+\left(5m-8\right)c = 0$ $\Rightarrow (5m-1)(m-2) = 0 \Rightarrow m = ...$ Or similar work with $c = 0$ throughout.	**M1**	2.1
	$\left(5m-8 \neq 0 \Rightarrow c = 0\right)$ Hence $m = 2$ gives an invariant line (with $c = 0$), so $y = 2x$ is invariant.	**A1**	1.1b
	Notes		
	M1 — Attempts to find the equation of a general invariant line, or general invariant line through the origin (so may have $c = 0$ throughout). To gain the method mark they must progress from finding the simultaneous equations to forming a quadratic in m and solving to a value of m.		
	A1 — Correct quadratic in m found, with $m = 2$ as solution (ignore the other) and deduction that hence $y = 2x$ is an invariant line. Ignore errors in the $(5m – 8)$ here as $c = 0$ is always a possible solution. No need to see $c = 0$ derived.		