Question	Scheme	Marks	AOs
3	$n=1,\ \sum_{r=1}^{1}\frac{1}{(2r-1)(2r+1)}=\frac{1}{1\times 3}=\frac{1}{3}$ and $\frac{n}{2n+1}=\frac{1}{2\times 1+1}=\frac{1}{3}$ (true for n=1)	B1	2.2a
	Assume general statement is true for $n=k$. So assume $\sum_{r=1}^{k}\frac{1}{(2r-1)(2r+1)}=\frac{k}{2k+1}$ is true.	M1	2.4
	$\left(\sum_{r=1}^{k+1}\frac{1}{(2r-1)(2r+1)}=\right)$ "$\frac{k}{2k+1}$" $+\frac{1}{(2k+1)(2k+3)}$	M1	2.1
	$=\frac{k(2k+3)+1}{(2k+1)(2k+3)}$	dM1	1.1b
	$=\frac{2k^2+3k+1}{(2k+1)(2k+3)}=\frac{(2k+1)(k+1)}{(2k+1)(2k+3)}=\frac{(k+1)}{2(k+1)+1}$ or $\frac{k+1}{2k+3}$	A1	1.1b
	As $\sum_{r=1}^{k+1}\frac{1}{(2r-1)(2r+1)}=\frac{(k+1)}{2(k+1)+1}$ then the general result is true for $n=k+1$ As the general result has been shown to be <u>true for $n=1$</u>, and <u>true for $n=k$ implies true for $n=k+1$</u>, so the result <u>is true for all n</u> $\in\mathbb{N}$	A1cso	2.4
		(6)	
			(6 marks)

Notes

B1	Substitutes $n=1$ into both sides of the statement to show they are eqaul. As a minimum expect to see $\frac{1}{1\times 3}$ and $\frac{1}{2+1}$ for the substitutions. (No need to state true for n = 1 for this mark.)
M1	Assumes (general result) true for $n=k$. (Assume (true for) n = k is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for n = k then ... etc.)
M1	Attempts to add $(k+1)$th term to their sum of k terms. Must be adding the (k+1)th term but allow slips with the sum.
dM1	Depends on previous M. Combines their two fractions over a correct common denominator for their fractions, which may be $(2k+1)^2(2k+3)$ (allow a slip in the numerator).
A1	Correct algebraic work leading to $\frac{(k+1)}{2(k+1)+1}$ or $\frac{k+1}{2k+3}$
A1	**cso** Depends on all except the **B** mark being scored (but must have an attempt to show the n = 1 case). Demonstrates the expression is the correct for n = k + 1 (both sides must have been seen somewhere) and gives a correct induction statement with **all** three underlined statements (or equivalents) seen at some stage during their solution (so true for n = 1 may be seen at the start). For demonstrating the correct expression, accept giving in the form $\frac{(k+1)}{2(k+1)+1}$, or reaching $\frac{k+1}{2k+3}$ and stating "which is the correct form with n = k + 1" or similar – but some indication is needed. Note: if mixed variables are used in working (r's and k's mixed up) then withhold the final A. Note: If n is used throughout instead of k allow all marks if earned.