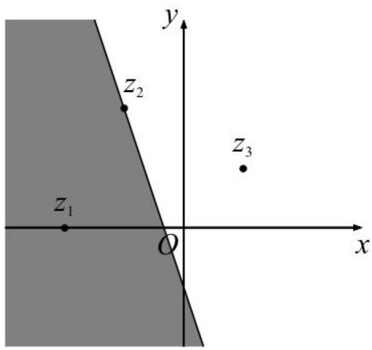
Question	Scheme	Marks	AOs
5 (a)	Complex roots of a real polynomial occur in conjugate pairs	M1	1.2
	so a polynomial with z_1, z_2 and z_3 as roots also needs z_2* and z_3* as roots, so 5 roots in total, but a quartic has at most 4 roots, so no quartic can have z_1, z_2 and z_3 as roots.	A1	2.4
		(2)	
(b)	$\dfrac{z_2 - z_1}{z_3 - z_1} = \dfrac{-1+2\mathrm{i}-(-2)}{1+\mathrm{i}-(-2)} = \dfrac{1+2\mathrm{i}}{3+\mathrm{i}} \times \dfrac{3-\mathrm{i}}{3-\mathrm{i}} = \ldots$	M1	1.1b
	$= \dfrac{3-\mathrm{i}+6\mathrm{i}+2}{9+1} = \dfrac{5+5\mathrm{i}}{10} = \dfrac{1}{2}+\dfrac{1}{2}\mathrm{i}$ oe	A1	1.1b
	As $\dfrac{1}{2}+\dfrac{1}{2}\mathrm{i}$ is in the first quadrant (may be shown by diagram), hence $\arg\left(\dfrac{z_2 - z_1}{z_3 - z_1}\right) = \arctan\left(\dfrac{1/2}{1/2}\right)\left(= \arctan(1)\right) = \dfrac{\pi}{4}$ *	A1*	2.1
		(3)	
(c)	$\arg\left(\dfrac{z_2 - z_1}{z_3 - z_1}\right) = \arg(z_2 - z_1) - \arg(z_3 - z_1) = \arg(1+2\mathrm{i}) - \arg(3+\mathrm{i})$	M1	1.1b
	Hence $\arctan(2) - \arctan\left(\dfrac{1}{3}\right) = \dfrac{\pi}{4}$ *	A1*	2.1
		(2)	
(d)	 Line passing through z_2 and the negative imaginary axis drawn.	B1	1.1b
	Area below and left of their line shaded, where the line must have negative gradient passing through negative imaginary axis but need not pass through z_2	B1	1.1b
	Unless otherwise indicated by the student mark Diagram 1(if used) if there are multiple attempts.		
		(2)	
			(9 marks)

Notes		
(a)	**M1**	Some evidence that complex roots occur as conjugate pairs shown, e.g. stated as in scheme, or e.g. identifying if $-1+2\text{i}$ is a root then so is $-1-2\text{i}$. Mere mention of complex conjugates is sufficient for this mark.
	A1	A complete argument, referencing that a quartic has at most 4 roots, but would need at least 5 for all of z_1, z_2 and z_3 as roots. There should be a clear statement about the number of roots of a quartic (e.g a quartic has four roots), and that this is not enough for the two conjugate pairs and real root.
(b)	**M1**	Substitutes the numbers in expression and attempts multiplication of numerator and denominator by the conjugate of their denominator or uses calculator to find the quotient. (May be implied.) NB Applying the difference of arguments and using decimals is M0 here.
	A1	Obtains $\frac{1}{2}+\frac{1}{2}\text{i}$. (May be from calculator.) Accepted equivalent Cartesian forms.
	A1*	Uses arctan on their quotient and makes reference to first quadrant or draws diagram to show they are in the first quadrant. to justify the argument.
(c)	**M1**	Applies the formula for the argument of a difference of complex numbers and substitutes values (may go directly to arctans if the arguments have already been established). If used in (b) it must be seen or referred to in (c) for this mark to be awarded. Allow for $\arg(z_2-z_1)-\arg(z_3-z_1)$ if z_2-z_1 and z_3-z_1 have been clearly identified in earlier work.
	A1*	Completes the proof clearly by identifying the required arguments and using the result of (b). Use of decimal approximations is A0.
(d)	**B1**	Draws a line through z_2 and passing through negative imaginary axis.
	B1	Correct side of bisector shaded. Allow this mark if the line does not pass through z_2. But it should be an attempt at the perpendicular bisector of the other two points – so have negative gradient and pass through the negative real axis. Ignore any other lines drawn for these two marks.