Question	Scheme	Marks	AOs
7. (a)	$\alpha+\beta+\left(\alpha+\frac{12}{\alpha}-\beta\right)=8$ so $2\alpha+\frac{12}{\alpha}=8$	M1	1.1b
		A1	1.1b
	$\Rightarrow 2\alpha^2-8\alpha+12=0$ or $\alpha^2-4\alpha+6=0$ $\Rightarrow \alpha=\frac{4\pm\sqrt{(-4)^2-4(1)(6)}}{2(1)}$ or $(\alpha-2)^2-4+6=0 \Rightarrow \alpha=...$	M1	1.1b
	$\Rightarrow \alpha=2\pm i\sqrt{2}$ are the two complex roots	A1	1.1b
	A correct full method to find the third root. Common methods are: Sum of roots = 8 $\Rightarrow$ third root $=8-\left(2+i\sqrt{2}\right)-\left(2-i\sqrt{2}\right)=...$ third root $=2+i\sqrt{2}+\frac{12}{2+i\sqrt{2}}-\left(2-i\sqrt{2}\right)=...$ Product of roots = 24 $\Rightarrow$ third root $=\frac{24}{\left(2+i\sqrt{2}\right)\left(2-i\sqrt{2}\right)}=...$ $(z-\alpha)(z-\beta)=z^2-4z+6 \Rightarrow f(z)=\left(z^2-4z+6\right)(z-\gamma)\Rightarrow \gamma=...$ (or long division to find third factor).	M1	3.1a
	Hence the roots of $f(z)=0$ are $2\pm i\sqrt{2}$ and 4	A1	1.1b
		(6)	
(b)	E.g. f(4) = 0 $\Rightarrow 4^3-8\times4^2+4p-24=0 \Rightarrow p=...$ Or $p=\left(2+i\sqrt{2}\right)\left(2-i\sqrt{2}\right)+4\left(2+i\sqrt{2}\right)+4\left(2-i\sqrt{2}\right)\Rightarrow p=...$ Or $f(z)=(z-4)(z^2-4z+6)\Rightarrow p=...$	M1	3.1a
	$\Rightarrow p=22$ **cso**	A1	1.1b
		(2)	
	(8 marks)		

Notes

(a)	**M1**	Equates sum of roots to 8 and obtains an equation in just α.
	A1	Obtains a correct equation in α.
	M1	Forms a three term quadratic equation in α and attempts to solve this equation by either completing the square or using the quadratic formula to give $\alpha=....$
	A1	$\alpha=2\pm i\sqrt{2}$
	M1	Any correct method for finding the remaining root. There are various routes possible. See scheme for common ones. Allow this mark if –24 is used as the product. See note below for a less common approach.
	A1	Third root found with all three roots correct. Note α and β need not be identified.
(b)	**M1**	Any correct method of finding p. For example, applies the factor theorem, process of finding the pair sum of roots, or uses the roots to form f(z).
	A1	$p=22$ by correct solution only. Note: this can be found using only their complex roots from (a) (e.g. by factor theorem)

Note for (a) final M – it is possible to find the second and third roots using only one initial root (e.g. if second root forgotten or error leads to only one initial root being found).

Product of roots = $\alpha\beta\left(\alpha+\frac{12}{\alpha}-\beta\right)=24 \Rightarrow \alpha\beta^2-\left(\alpha^2+12\right)\beta+24=0$, substitutes in α and attempts to solve the quadratic in β to achieve remaining roots. The final M can be gained once three roots in total have been obtained. (This is unlikely to be seen as part of a correct answer.) Allow if −24 has been used for the product.