Question	Scheme	Marks	AOs
9.	A correct overall strategy, an attempt at integrating y^2 with respect to x combine in some way with the volume of revolution formula (use of $\pi\int y^2\mathrm{d}x$ or $\alpha\int y^2\mathrm{d}x$ for any variable α is fine) followed by attempt to find an angle/form an equation in θ	M1	3.1a
	$y^2 = kx^{\frac{2}{3}} + \ldots + \dfrac{m}{x^{\frac{4}{3}}}$ or $y^2 = kx^{\frac{2}{3}} + \ldots + mx^{-\frac{4}{3}}$ where … is one or two more terms.	M1	1.1b
	$y^2 = 4x^{\frac{2}{3}} + 4x^{-\frac{1}{3}} + x^{-\frac{4}{3}}$ or $y^2 = 4x^{\frac{2}{3}} + 2x^{-\frac{1}{3}} + x^{-\frac{4}{3}} + 2x^{-\frac{1}{3}}$ (oe)	A1	1.1b
	$\int y^2\mathrm{d}x = \int 4x^{\frac{2}{3}} + \dfrac{4}{x^{\frac{1}{3}}} + \dfrac{1}{x^{\frac{4}{3}}}\mathrm{d}x = \alpha x^{\frac{5}{3}} + \beta x^{\frac{2}{3}} + \gamma x^{-\frac{1}{3}}$	M1	1.1b
	$= \dfrac{12x^{\frac{5}{3}}}{5} + 6x^{\frac{2}{3}} - \dfrac{3}{x^{\frac{1}{3}}}$ (oe)	A1ft A1	1.1b 1.1b
	$\dfrac{\theta}{2}\left[\dfrac{12x^{\frac{5}{3}}}{5} + 6x^{\frac{2}{3}} - \dfrac{3}{x^{\frac{1}{3}}}\right]_{\frac{1}{8}}^{8} = \dfrac{461}{2}$ $\Rightarrow \dfrac{\theta}{2}\left[\left(\dfrac{12\times 8^{\frac{5}{3}}}{5} + 6\times 8^{\frac{2}{3}} - \dfrac{3}{8^{\frac{1}{3}}}\right) - \left(\dfrac{12\times\left(\frac{1}{8}\right)^{\frac{5}{3}}}{5} + 6\times\left(\dfrac{1}{8}\right)^{\frac{2}{3}} - \dfrac{3}{\left(\frac{1}{8}\right)^{\frac{1}{3}}}\right)\right] = \dfrac{461}{2} \Rightarrow \theta = \ldots$ OR $\pi\left[\dfrac{12x^{\frac{5}{3}}}{5} + 6x^{\frac{2}{3}} - \dfrac{3}{x^{\frac{1}{3}}}\right]_{\frac{1}{8}}^{8} = \pi\left[\left(\dfrac{12\times 8^{\frac{5}{3}}}{5} + 6\times 8^{\frac{2}{3}} - \dfrac{3}{8^{\frac{1}{3}}}\right) - \left(\dfrac{12\times\left(\frac{1}{8}\right)^{\frac{5}{3}}}{5} + 6\times\left(\dfrac{1}{8}\right)^{\frac{2}{3}} - \dfrac{3}{\left(\frac{1}{8}\right)^{\frac{1}{3}}}\right)\right] = ..$ followed by $\dfrac{\theta}{2\pi}\times\ldots = \dfrac{461}{2} \Rightarrow \theta = \ldots$	M1	3.1a
	$\theta = \dfrac{40}{9}$ (radians)	A1	1.1b
		(8)	
	(8 marks)		

Notes		
	M1	A correct overall strategy, either finding full volume rotated by 2π first, then performing some kind of scaling, or using $\alpha\int y^2\,dx$ for a variable α (ideally $\frac{\theta}{2}$, but for the strategy accept with any variable multiple), to form an equation in just the angle.
	M1	Attempting to square y to a three or four term expression. Look for correct powers on first and last term with some term(s) in the middle.
	A1	Correct expansion in three or four terms – award when first seen.
	M1	Integrates y^2 w.r.t. x. Must have at least two terms in their y^2 with fractional indices. Power to be increased by 1 in at least two terms.
	A1ft	Two terms of integral correct. Follow through on their expansion. Need not be simplified.
	A1	Fully correct integral. Need not be simplified. May still be four terms
	M1	Either : Substitutes limits and subtracts correct way round (must be seen or implied by the answer), and equates to $\frac{461}{2}$ if using $\frac{1}{2}\theta\int y^2\,dx$ and proceeds to find θ. Or : Substitutes limits and subtracts correct way round (seen or implied) and multiplies by π to get the full volume AND then multiplies the result by $\frac{\theta}{2\pi}$ before equating to $\frac{461}{2}$. **The method must be correct for this mark – so they must be using** $\frac{\theta}{2}\int y^2\,dx$ **directly or** $\pi\int y^2\,dx$ **and scale by** $\frac{\theta}{2\pi}$ **when setting equal to** $\frac{461}{2}$
	A1	Correct angle found. Accept $\frac{40}{9}$, awrt 4.44 or awrt $255°$ (as long as the degrees units are made clear – do not accept just 255) isw once a correct value of θ is found.

Special case The question specified that algebraic integration must be used, so use of a calculator to find the integral cannot score the marks for integration but may be allowed the strategy and answer marks. A maximum of M1M0A0M0A0A0M1A1 is available in such cases.

Expanding y^2 first but showing no integration can score the second M and first A (if earned) as well.

Note that $\int_{1/8}^{8}\left(2x^{1/3}+x^{-2/3}\right)^2\,dx=\frac{4149}{40}=103.725$ but just this alone is worth **no marks**. There must be an attempt to incorporate this within a strategy to gain access to marks.