Question	Scheme	Marks	AOs
10. (a)	a represents the proportion of juvenile chimpanzees that (survive and) **remain** juvenile chimpanzees the next year.	B1	3.4
		(1)	
(b)(i)	Determinant $= 0.82a - 0.08\times 0.15$	M1	1.1b
	$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1} = \ldots \begin{pmatrix} 0.82 & -0.15 \\ -0.08 & a \end{pmatrix}$	M1	1.1b
	$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1} = \dfrac{1}{0.82a - 0.012}\begin{pmatrix} 0.82 & -0.15 \\ -0.08 & a \end{pmatrix}$	A1	1.1b
		(3)	
(ii)	$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1}\begin{pmatrix} 15360 \\ 43008 \end{pmatrix} = \dfrac{1}{0.82a - 0.012}\begin{pmatrix} 0.82\times 15360 - 0.15\times 43008 \\ (-0.08)\times 15360 + 43008a \end{pmatrix}$ OR forms equations $\begin{array}{l} 15360 = aJ_0 + 0.15\times A_0 \\ 43008 = 0.08\times J_0 + 0.82\times A_0 \end{array}$	M1	3.1a
	$\dfrac{1}{0.82a - 0.012}\left[6144 + \left(43008a - 1228.8\right)\right] = 64000$ $\Rightarrow 4915.2 + 43008a = 64000(0.82a - 0.012) \Rightarrow a = \ldots$ OR $A_0 = 64000 - J_0 \Rightarrow 43008 = 0.08\times J_0 + 0.82\times\left(64000 - J_0\right) = J_0 = \ldots$ $\Rightarrow a = \dfrac{15360 - \left(64000 - J_0\right)}{J_0} = \ldots$	M1	3.1a
	$a = \dfrac{5683.2}{9472} = 0.60$	A1	1.1b
		(3)	
(iii)	Initial juvenile population $= \dfrac{"6144"}{"0.48"} = 12800$	M1	3.4
	So change of 2560 juvenile chimpanzees	A1	1.1b
		(2)	
(c)	As the number of juveniles has increased, the model is not initially predicting a decline, so is not suitable in the short term. (Follow through their answer to (b) – but they must have made an attempt at it to find at least a value for J_0)	B1ft	3.5a
		(1)	
(d)	Third category needs to be introduced for chimpanzees aged 40 and above, mature chimpanzees M_n, and a matrix multiplication of increased dimension set up. Accept $3\times 3, 3\times 2$ or 2×3 matrices including all three categories in the column vector.	M1	3.5c
	The corresponding matrix model will have the form $\begin{pmatrix} J_{n+1} \\ A_{n+1} \\ M_{n+1} \end{pmatrix} = \begin{pmatrix} a & b & \underline{\mathbf{0}} \\ 0.08 & c & 0 \\ 0 & d & e \end{pmatrix}\begin{pmatrix} J_n \\ A_n \\ M_n \end{pmatrix}$ (The underlined zero must be correct but do not be concerned about any values used in the other entries.)	A1	3.3
		(2)	
	(12 marks)		

Notes		
(a)	**B1**	Correct interpretation. Need not mention survival but must be clear it is the (proportion of) **juveniles that remain as juveniles** the next year (ie those that survive but don't progress to adulthood). E.g. accept "(number of) juveniles who do not become adults" but do not accept "surviving juveniles".
		Mark part (b) as a whole.
(b)(i)	**M1**	Attempts the determinant in terms of a Allow miscopies for the attempt. Allow $0.82a - 0.12$ as a slip.
	M1	Attempts the form of the inverse, swapped leading diagonals and sign changed on both off diagonals. Allow miscopies of the numbers but the signs must be correct.
	A1	Correct inverse matrix
(ii)	**M1**	Use the inverse matrix and attempts to find the initial juvenile and adult populations. . (May have determinant 1 for this mark.) Alternatively, sets up simultaneous equations from the original system, $15360 = aJ_0 + 0.15\times A_0$ and $43008 = 0.08\times J_0 + 0.82\times A_0$ Accept with J_n and A_n or other appropriate variables.
	M1	Uses the sum of initial populations equals 64000 in an attempt to find a. (May have determinant 1 for this mark.) If using alternative, use of e.g. $A_0 = 64000 - J_0$ in second equation to find J_0, followed by attempt to find a. Award for an attempt to solve the equations, but don't be too concerned with the algebraic process as long as they are attempting to use all three equations.
	A1	Correct value, $a = 0.6$ (or 0.60 or $\frac{3}{5}$).
(iii)	**M1**	Uses their a to find the value of J_0. This mark may be gained for work done in (ii) if the alternative has been used but must have come from a correct method.
	A1	Correct difference found, as long as there is no contradictory statement – so "decrease of 2560" is A0.
(c)	**B1ft**	Comments that the change is an increase so does not fit the model. Follow through their answer to (b) as long as at least a value for J_0 has been found. If a decrease has been found allow for commenting the model is suitable. If an answer is given to (b)(iii), follow through on whatever their answer is. If no answer has been given, but an initial population found, a comparison should be made between this value and 153600 with conclusion must be consistent with their answer for J_0
(d)	**M1**	Introduces a third category (may be *M*ature, *E*lderly or any suitable letter used) and sets up a matrix multiplication (the left hand side may be missing for this mark) with all three categories in the column vector. The dimension of the matrix should be 3 in at least either row or column, and there should be a 3×1 vector.
	A1	Sets up the new matrix equation, including both sides and making clear the zero (underlined) so that the correct progression that no new juveniles arise from the mature chimpanzees is clear. Overlook other values, though ideally the other two zeroes are shown too, to indicate mature chimpanzees do not regress to adulthood, and juveniles cannot proceed directly to mature chimpanzees.