5.

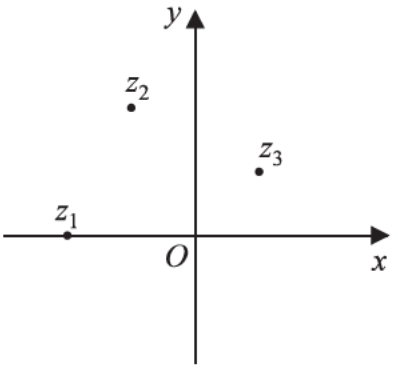

Figure 1

The complex numbers $z_1 = -2$, $z_2 = -1 + 2\mathrm{i}$ and $z_3 = 1 + \mathrm{i}$ are plotted in Figure 1, on an Argand diagram for the complex plane with $z = x + \mathrm{i}y$

(a) Explain why z_1, z_2 and z_3 cannot all be roots of a quartic polynomial equation with real coefficients.

(2)

(b) Show that $\arg\left(\dfrac{z_2 - z_1}{z_3 - z_1}\right) = \dfrac{\pi}{4}$

(3)

(c) Hence show that $\arctan(2) - \arctan\left(\dfrac{1}{3}\right) = \dfrac{\pi}{4}$

(2)

A copy of Figure 1, labelled Diagram 1, is given on page 12.

(d) Shade, on Diagram 1, the set of points of the complex plane that satisfy the inequality

$$|z + 2| \leqslant |z - 1 - \mathrm{i}|$$

(2)

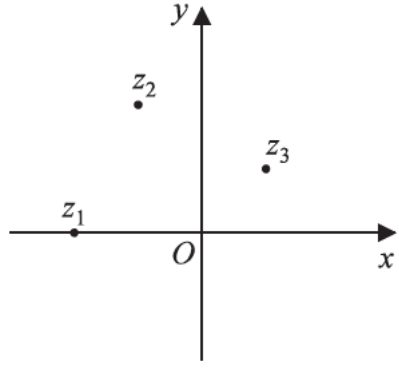

Diagram 1