8. A gas company maintains a straight pipeline that passes under a mountain.

The pipeline is modelled as a straight line and one side of the mountain is modelled as a plane.

There are accessways from a control centre to two access points on the pipeline.

Modelling the control centre as the origin O, the two access points on the pipeline have coordinates $P(-300, 400, -150)$ and $Q(300, 300, -50)$, where the units are metres.

(a) Find a vector equation for the line PQ, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where λ is a scalar parameter.

(2)

The equation of the plane modelling the side of the mountain is $2x + 3y - 5z = 300$

The company wants to create a new accessway from this side of the mountain to the pipeline.

The accessway will consist of a tunnel of shortest possible length between the pipeline and the point $M(100, k, 100)$ on this side of the mountain, where k is a constant.

(b) Using the model, find

(i) the coordinates of the point at which this tunnel will meet the pipeline,

(ii) the length of this tunnel.

(7)

It is only practical to construct the new accessway if it will be significantly shorter than both of the existing accessways, OP and OQ.

(c) Determine whether the company should build the new accessway.

(2)

(d) Suggest one limitation of the model.

(1)