

Question	Scheme	Marks	AOs
2(a)	$ z_1 = \sqrt{13}$ and $\arg z_1 = \tan^{-1}\left(\frac{3}{2}\right)$	B1	1.1b
	$z_1 = \sqrt{13}(\cos 0.9828 + i \sin 0.9828)$	B1ft	1.1b
		(2)	
(b)	A complete method to find the modulus of z_2 e.g. $ z_1 = \sqrt{13}$ and uses $ z_1 z_2 = z_1 \times z_2 = 39\sqrt{2} \Rightarrow z_2 = 3\sqrt{26}$ or $\sqrt{234}$	M1 A1	3.1a 1.1b
	A complete method to find the argument of z_2 e.g. $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) = \frac{\pi}{4} \Rightarrow \arg(z_2) = \dots$ $\arg(z_2) = \frac{\pi}{4} - \tan^{-1}\left(\frac{3}{2}\right)$ or $\frac{\pi}{4} - 0.9828$ or $-0.1974\dots$	M1 A1	3.1a 1.1b
	$z_2 = 3\sqrt{26}(\cos(' - 0.1974\dots') + i \sin(' - 0.1974\dots'))$ or $z_2 = a + bi \Rightarrow a^2 + b^2 = 234$ and $\tan^{-1}(-0.1974) = \frac{b}{a} \Rightarrow \frac{b}{a} = -0.2$ $\Rightarrow a = \dots$ and $b = \dots$	ddM1	1.1b
	Deduces that $z_2 = 15 - 3i$ only	A1	2.2a
	Alternative $z_1 z_2 = (a + bi)(2 + 3i) = (2a - 3b) + (3a + 2b)i$		
	$(2a - 3b)^2 + (3a + 2b)^2 = (39\sqrt{2})^2$ or 3042 $\Rightarrow a^2 + b^2 = 234$ or $ z_1 z_2 = z_1 \times z_2 = 39\sqrt{2} \Rightarrow z_2 = 3\sqrt{26}$ or $\sqrt{234}$ $\Rightarrow a^2 + b^2 = 234$	M1 A1	3.1a 1.1b
	$\arg[(2a - 3b) + (3a + 2b)i] = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{3a + 2b}{2a - 3b}\right) = \frac{\pi}{4} \Rightarrow \frac{3a + 2b}{2a - 3b} = 1$ $\Rightarrow a = -5b$	M1 A1	3.1a 1.1b
	Solves $a = -5b$ and $a^2 + b^2 = 234$ to find values for a and b	ddM1	1.1b
	Deduces that $z_2 = 15 - 3i$ only	A1	2.2a
		(6)	

(8 marks)

Notes:**(a)**

B1: Correct exact value for $|z_1| = \sqrt{13}$ and $\arg z_1 = \tan^{-1}\left(\frac{3}{2}\right)$. The value for $\arg z_1$ can be implied by sight of awrt 0.98 or awrt 56.3°

B1ft: Follow through on $r = |z_1|$ and $\theta = \arg z_1$ and writes $z_1 = r(\cos \theta + i \sin \theta)$ where r is exact and θ is correct to 4 s.f. do not follow through on rounding errors.

(b)

M1: A complete method to find the modulus of z_2

A1: $|z_2| = 3\sqrt{26}$

M1: A complete method to find the argument of z_2

A1: $\arg(z_2) = \frac{\pi}{4} - \tan^{-1}\left(\frac{3}{2}\right)$ or $\frac{\pi}{4} - 0.9828$ or $-0.1974\dots$

ddM1: Writes z_2 in the form $r(\cos \theta + i \sin \theta)$, dependent on both previous M marks.

Alternative forms two equations involving a and b using the modulus and argument of z_2 and solve to find values for a and b

A1: Deduces that $z_2 = 15 - 3i$ only

(b) Alternative: $z_1 z_2 = (a + bi)(2 + 3i) = (2a - 3b) + (3a + 2b)i$

M1: A complete method to find an equation involving a and b using the modulus

A1: Correct simplified equation $a^2 + b^2 = 234$ o.e.

M1: A complete method to find an equation involving a and b using the argument.

Note $\tan^{-1}\left(\frac{2a - 3b}{3a + 2b}\right) = \frac{\pi}{4}$ this would score **M0 A0 ddM0 A0**

A1: Correct simplified equation $a = -5b$ o.e.

ddM1: Dependent on both the previous method marks. Solves their equations to find values for a and b

A1: Deduces that $z_2 = 15 - 3i$ only