

Question	Scheme	Marks	AOs
6(i) (a)	<p>Multiplies the matrix A by itself and sets equal to I to form one equation in a only and another equation involving both a and b.</p> $\begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix} \begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow 4 + a(a-4) = 1$ <p>and either $2a + ab = 0$ or $2(a-4) + b(a-4) = 0$ or $a(a-4) + b^2 = 1$</p>	M1	3.1a
	<p>Solves a 3TQ involving only the constant a. This could come after a value of b is found and this value substituted into an equation involving both a and b</p> $a^2 - 4a + 3 = 0 \Rightarrow (a-3)(a-1) = 0 \Rightarrow a = \dots$	dM1	1.1b
	$a = 1, a = 3$	A1	11b
	<p>Substitutes a value for a into an equation involving both a and b and solves for b.</p> <p>e.g.</p> $2(1) + (1)b \Rightarrow b = \dots$ $2(1-4)b + (1-4) = 0 \Rightarrow b = \dots$ $(1)(1-4) + b^2 = 1 \Rightarrow b = \dots$	dM1	1.1b
	<p>Alternatively uses</p> $2a + ab = 0$ $a(2+b) = 0$ <p>As $a \neq 0$ $2+b=0 \Rightarrow b = \dots$</p>		
	$b = -2$	A1	1.1b
		(5)	
	<p style="text-align: center;">Alternative (i) (a)</p> <p>Finds \mathbf{A}^{-1} in terms of a and b, sets equal to \mathbf{A} and attempts to find at least two different equations. Allow a single sign slip</p> $\frac{1}{2b - a(a-4)} \begin{pmatrix} b & -a \\ -(a-4) & 2 \end{pmatrix} = \begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix}$ <p>One equation from $\frac{b}{2b - a(a-4)} = 2, \frac{2}{2b - a(a-4)} = b$</p> <p>One equation from $\frac{-a}{2b - a(a-4)} = a, \frac{-(a-4)}{2b - a(a-4)} = a-4$</p>	M1	3.1a
	<p>Uses their value of b and their value of the determinant to form and solve a 3TQ involving only the constant a</p> $a^2 - 4a + 3 = 0$ $\Rightarrow (a-3)(a-1) = 0$ $\Rightarrow a = \dots$	dM1	1.1b
	$a = 1, a = 3$	A1	1.1b

	$\frac{-a}{2b-a(a-4)} = a$ $\Rightarrow 2b-a(a-4) = -1 \Rightarrow \frac{b}{-1} = 2$ <p style="text-align: center;">Or</p> $\frac{-(a-4)}{2b-a(a-4)} = a-4$ $\Rightarrow 2b-a(a-4) = -1$ $\Rightarrow \frac{2}{-1} = b$	Substitutes a value for a into an equation to find a value for b	dM1	1.1b
	$b = -2$		A1	1.1b
(b)	<p>Uses their smallest value of \mathbf{a} and their value for \mathbf{b} to form two equations</p> $\begin{pmatrix} 2 & 'a' \\ 'a-4' & 'b' \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow 2x+ay = x \text{ and } (a-4)x+by = y$ $\begin{pmatrix} 2 & 1 \\ -3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow 2x+y = x \text{ and } -3x-2y = y$		M1	3.1a
	$2x+y = x \Rightarrow x+y = 0$ o.e. and $-3x-2y = y \Rightarrow x+y = 0$ o.e.		M1	1.1b
	$x+y = 0$ o.e.		A1	2.1
			(3)	
(ii)(a)	Area of the triangle $T = 3$		B1	1.1b
	<p>Complete method to find a value for p. Need to see an attempt at the determinant and setting equal to 15 divided by their area of T. The resulting 3TQ needs to be solved to find a value of p.</p> <p>Determinant $3p \times p - (-1) \times 2p = \frac{15}{\text{'their area'}} \Rightarrow p = \dots$</p>		M1	3.1a
	$3p^2 + 2p - 5 (= 0)$		A1	1.1b
	$p = 1$ must reject $p = -\frac{5}{3}$		A1	1.1b
			(4)	
(b)	$\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$		B1 B1	1.1b 1.1b
			(2)	
(c)	<p>(their matrix found in part (b)) $\begin{pmatrix} 'p' & 2'p' \\ -1 & 3'p' \end{pmatrix} = \begin{pmatrix} \dots & \dots \\ \dots & \dots \end{pmatrix}$</p> $\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} \dots & \dots \\ \dots & \dots \end{pmatrix}$		M1	1.1b

$$\begin{pmatrix} 3 & 6 \\ 2 & -6 \end{pmatrix}$$

A1ft

1.1b

(2)

(16 marks)

Notes:**(i)(a)**

M1: Forming two equations, one involving a only and one involving a and b

dM1: Dependent on previous mark, solves a 3TQ involving a

A1: Correct values for a

dM1: Dependent on first method mark Substitutes one of their values of a into an equation involving a and b and solve to find a value for b . Alternatively factorises either $2a + ab = 0$ and uses $a \neq 0$ to find a value for b .

A1: Correct value for b

Alternative(i)(a)

M1: Finds \mathbf{A}^{-1} and sets equal to \mathbf{A} and forms two different equations

dM1: Dependent on previous mark. Eliminates b from their equations and solves a 3TQ involving only the constant a . Alternatively if the value of b is found first substitutes their value for b into their determinant $= -1$ to form and solve a 3TQ for a

A1: Correct value for a

dM1: Dependent on first method mark. Substitutes a value for a into an equation to find a value for b . Alternatively uses one equation to find the determinant $= -1$ and uses this to find a value of b .

A1: Correct values for b

(b)

M1: Extracts simultaneous equations using their matrix \mathbf{A} with their smaller value of a .

M1: Gathers terms from their two equations.

A1: Achieves the correct equations and deduces the correct line. Accept equivalent equations as long as both have been shown to be the same.

(ii)(a)

B1: Area of the triangle $T = 3$

M1: Full method. Finds the determinant, sets equal to 15/their area and solves the resulting 3TQ

A1: Correct quadratic

A1: $p = 1$ only

(b)

B1 One correct row or column

B1: All correct

(c)

M1: Multiplies the matrices \mathbf{QP} in the correct order (if answer only then evidence can be taken from 3 correct or 3 correct ft elements)

A1ft: Correct matrix following through on their answer to part (b) and their value of p as long as it is a positive constant