Question	Scheme	Marks	AOs
8	**Way 1: $\mathbf{f}(\boldsymbol{k}+1)-\mathbf{f}(\boldsymbol{k})$** When $n=1$, $2^{n+2}+3^{2n+1}=2^3+3^3=35$ Shows the statement is true for $n=1$, allow 5(7)	B1	2.2a
	Assume true for $\boldsymbol{n}=\boldsymbol{k}$, so $2^{k+2}+3^{2k+1}$ is divisible by 7	M1	2.4
	$\mathrm{f}(k+1)-\mathrm{f}(k)=2^{k+3}+3^{2k+3}-\left(2^{k+2}+3^{2k+1}\right)$	M1	2.1
	$=2\times2^{k+2}+9\times3^{2k+1}-2^{k+2}-3^{2k+1}$ $=2^{k+2}+8\times3^{2k+1}$ $=\mathrm{f}(k)+7\times3^{2k+1}$ or $8\mathrm{f}(k)-7\times2^{k+2}$	A1	1.1b
	$\mathrm{f}(k+1)=2\mathrm{f}(k)+7\times3^{2k+1}$ or $9\mathrm{f}(k)-7\times2^{k+2}$	A1	1.1b
	If **true for $\boldsymbol{n}=\boldsymbol{k}$** then **true for $\boldsymbol{n}=\boldsymbol{k}+1$** and as it is **true for $\boldsymbol{n}=1$** the statement is **true for all** (positive integers) $\boldsymbol{n}$	A1	2.4
		(6)	
	Way 2: $\mathbf{f}(\boldsymbol{k}+1)$ When $n=1$, $2^{n+2}+3^{2n+1}=2^3+3^3=35$ So the statement is true for $n=1$	B1	2.2a
	Assume true for $n=k$, so $2^{k+2}+3^{2k+1}$ is divisible by 7	M1	2.4
	$\mathrm{f}(k+1)=2^{(k+1)+2}+3^{2(k+1)+1}$	M1	2.1
	$\mathrm{f}(k+1)=2^{k+3}+3^{2k+3}=2\times2^{k+2}+9\times3^{2k+1}$ $=2\left(2^{k+2}+3^{2k+1}\right)+7\times3^{2k+1}$ $=2\mathrm{f}(k)+7\times3^{2k+1}$ or $9\mathrm{f}(k)-7\times2^{k+2}$	A1 A1	1.1b 1.1b
	If true for $\boldsymbol{n}=\boldsymbol{k}$ then **true for $\boldsymbol{n}=\boldsymbol{k}+1$** and as it is **true for $\boldsymbol{n}=1$** the statement is **true for all** (positive integers) $\boldsymbol{n}$	A1	2.4
		(6)	
	Way 3: $\mathbf{f}(\boldsymbol{k}+1)-\boldsymbol{m}\,\mathbf{f}(\boldsymbol{k})$ When $n=1$, $2^{n+2}+3^{2n+1}=2^3+3^3=35$ So the statement is true for $n=1$	B1	2.2a
	Assume true for $n=k$, so $2^{k+2}+3^{2k+1}$ is divisible by 7	M1	2.4
	$\mathrm{f}(k+1)-m\mathrm{f}(k)=2^{k+3}+3^{2k+3}-m\left(2^{k+2}+3^{2k+1}\right)$	M1	2.1
	$=2\times2^{k+2}+9\times3^{2k+1}-m\times2^{k+2}-m\times3^{2k+1}$ $=(2-m)2^{k+2}+9\times3^{2k+1}-m\times3^{2k+1}$ $=(2-m)\left(2^{k+2}+3^{2k+1}\right)+7\times3^{2k+1}$	A1	1.1b
	$\mathrm{f}(k+1)=(2-m)\left(2^{k+2}+3^{2k+1}\right)+7\times3^{2k+1}+m\mathrm{f}(k)$	A1	1.1b

	If true for $\boldsymbol{n=k}$ then **true for** $\boldsymbol{n=k}+1$ and as it is **true for** $\boldsymbol{n}=1$ the statement is **true for all** (positive integers) $\boldsymbol{n}$	A1	2.4
		(6)	

(6 marks)

Notes:

Way 1: $\mathbf{f}(\boldsymbol{k}+1)-\mathbf{f}(\boldsymbol{k})$

B1: Shows that f(1) = 35 and concludes or shows divisible by 7. This may be seen in the final statement.

M1: Makes a statement that assumes the result is true for some value of *n*

M1: Attempts $\mathrm{f}(k+1)-\mathrm{f}(k)$

A1: Achieves a correct expression for $\mathrm{f}(k+1)-\mathrm{f}(k)$ in terms of $\mathrm{f}(k)$

A1: Reaches a correct expression for $\mathrm{f}(k+1)$ in terms of $\mathrm{f}(k)$

A1: Correct conclusion. This mark is dependent on all previous marks, look out for scoring B1 in this statement. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution.

Way 2: $\mathbf{f}(\boldsymbol{k}+1)$

B1: Shows that f(1) = 35 and concludes divisible by 7

M1: Makes a statement that assumes the result is true for some value of *n*

M1: Attempts $\mathrm{f}(k+1)$

A1: Correctly obtains either $2\mathrm{f}(k)$ **or** $7\times3^{2k+1}$ or either $9\mathrm{f}(k)$ or $-7\times2^{k+2}$

A1: Reaches a correct expression for $\mathrm{f}(k+1)$ in terms of $\mathrm{f}(k)$

A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution.

Way 3: $\mathbf{f}(\boldsymbol{k}+1)-\boldsymbol{m}\,\mathbf{f}(\boldsymbol{k})$

B1: Shows that f(1) = 35 and concludes divisible by 7

M1: Makes a statement that assumes the result is true for some value of *n*

M1: Attempts $\mathrm{f}(k+1)-m\mathrm{f}(k)$

A1: Achieves a correct expression for $\mathrm{f}(k+1)-m\mathrm{f}(k)$ in terms of $\mathrm{f}(k)$

A1: Reaches a correct expression for $\mathrm{f}(k+1)$ in terms of $\mathrm{f}(k)$

A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution.