Question	Scheme	Marks	AOs
8(a)	$n=1,\quad \text{lhs}=1(2)(3)=6,\quad \text{rhs}=\frac{1}{2}(1)(2)^2(3)=6$ (true for $n = 1$)	B1	2.2a
	Assume true for $n = k$ so $\sum_{r=1}^{k} r(r+1)(2r+1)=\frac{1}{2}k(k+1)^2(k+2)$	M1	2.4
	$\sum_{r=1}^{k+1} r(r+1)(2r+1)=\frac{1}{2}k(k+1)^2(k+2)+(k+1)(k+2)(2k+3)$	M1	2.1
	$=\frac{1}{2}(k+1)(k+2)\left[k(k+1)+2(2k+3)\right]$	dM1	1.1b
	$=\frac{1}{2}(k+1)(k+2)\left[k^2+5k+6\right]=\frac{1}{2}(k+1)(k+2)(k+2)(k+3)$ Shows that $=\frac{1}{2}(\underline{k+1})(\underline{k+1}+1)^2(\underline{k+1}+2)$ Alternatively shows that $\sum_{r=1}^{k+1} r(r+1)(2r+1)=\frac{1}{2}(k+1)(k+1+1)^2(k+1+2)$ $=\frac{1}{2}(k+1)(k+2)^2(k+3)$ Compares with their summation and concludes true for $n = k+1$, may be seen in the conclusion.	A1	1.1b
	If the statement is **true for *n* = *k* then** it has been shown **true for *n* = *k* + 1** and as it is **true for *n* = 1**, the statement is **true** for all **positive integers *n*.**	A1	2.4
		(6)	
(b)	$\sum_{r=n}^{2n} r(r+1)(2r+1)=\frac{1}{2}(2n)(2n+1)^2(2n+2)-\frac{1}{2}(n-1)n^2(n+1)$	M1	3.1a
	$=\frac{1}{2}n(n+1)\left[4(2n+1)^2-n(n-1)\right]$	M1	1.1b
	$=\frac{1}{2}n(n+1)\left(15n^2+17n+4\right)$ $=\frac{1}{2}n(n+1)(3n+1)(5n+4)$	A1	1.1b
		(3)	
			(9 marks)

Notes
(a) Note ePen B1 M1 M1 A1 A1 A1 B1: Substitutes n = 1 into both sides to show that they are both equal to 6. (There is no need to state true for n = 1 for this mark) M1: Makes a statement that assumes the result is true for some value of n, say k M1: Adds the (k + 1)th term to the assumed result dM1: Dependent on previous M, factorises out $\frac{1}{2}(k+1)(k+2)$ A1: Reaches a correct the required expression no errors and shows that this is the correct sum for $n = k +1$ A1: Depends on all except **B** mark being scored (must have been some attempt to show true for n = 1). Correct conclusion conveying all the points in bold. (b) M1: Realises that $\sum_{r=1}^{2n} r(r+1)(2r+1) - \sum_{r=1}^{n-1} r(r+1)(2r+1)$ is required and uses the result from part (a) to obtain the required sum in terms of n M1: Attempts to factorise by $\frac{1}{2}n(n+1)$ A1: Correct expression or correct values