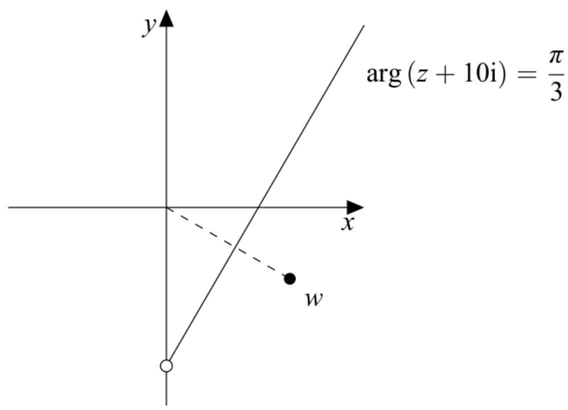
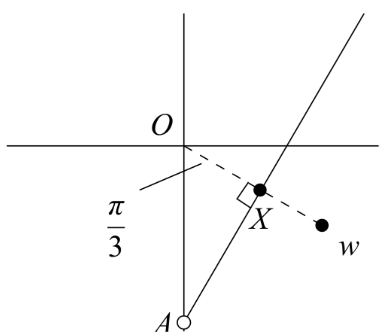
Question	Scheme	Marks	AOs
2(a)	$\lvert w\rvert=\sqrt{\left(4\sqrt{3}\right)^2+(-4)^2}=8$	B1	1.1b
	$\arg w=\arctan\left(\dfrac{\pm4}{4\sqrt{3}}\right)=\arctan\left(\pm\dfrac{1}{\sqrt{3}}\right)$	M1	1.1b
	$=-\dfrac{\pi}{6}$	A1	1.1b
	So $(w=)8\left(\cos\left(-\dfrac{\pi}{6}\right)+\mathrm{i}\sin\left(-\dfrac{\pi}{6}\right)\right)$	A1	1.1b
		(4)	

(b)



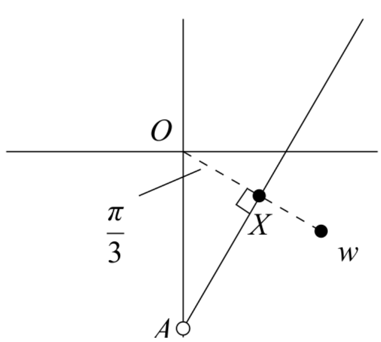
Question	Scheme	Marks	AOs
(b)	(i) w in 4th quadrant with either $\left(4\sqrt{3},-4\right)$ seen or $-\dfrac{\pi}{4}<\arg w<0$	B1	1.1b
	(ii) half line with positive gradient emanating from imaginary axis.	M1	1.1b
	The half line should pass between O and w starting from a point on the imaginary axis below w	A1	1.1b
		(3)	

(c)



Question	Scheme	Marks	AOs
(c)	ΔOAX is right angled at X so $OX=10\sin\dfrac{\pi}{6}=5$ (oe)	M1	3.1a
	So shortest distance is $WX=OW-OX=$ '8' $-5=\ldots$	M1	1.1b
	So min distance is 3	A1	1.1b

Alternative 1



Question	Scheme	Marks	AOs
	A complete method to find the coordinates of X. Finds the equation of the line from O to w, $y=-\dfrac{1}{\sqrt{3}}x$ and the equation of the half line $y=\sqrt{3}x-10$, solves to find the point of intersection $X\left(\dfrac{5\sqrt{3}}{2},-\dfrac{5}{2}\right)$	M1	3.1a
	Finds the length WX $\sqrt{\left(4\sqrt{3}-\dfrac{5\sqrt{3}}{2}\right)^2+\left(-4--\dfrac{5}{2}\right)^2}$	M1	1.1b
	So min distance is 3	A1	1.1b
	Alternative 2	M1	3.1a

	Finds the length $AW = \sqrt{\left(4\sqrt{3}-0\right)^2 + \left(-4--10\right)^2} = ...\left\{\sqrt{84}\right\}$ Finds the angle between the horizontal and the line AW $= \tan^{-1}\left(\frac{-4--10}{4\sqrt{3}}\right) = ...\left\{0.7137...\text{radians or } 40.89...^{\circ}\right\}$		
	Finds the length of $WX = \sqrt{84}\times\sin\left(\frac{\pi}{3} - 0.7137\right) = ...$ Or $= \sqrt{84}\times\sin\left(60-40.89\right) = ...$	M1	1.1b
	So min distance is 3	A1	1.1b
	Alternative 3 Vector equation of the half line $r = \begin{pmatrix}0\\-10\end{pmatrix} + \lambda\begin{pmatrix}1\\\sqrt{3}\end{pmatrix}$ $XW = \begin{pmatrix}4\sqrt{3}-\lambda\\-4-\lambda\sqrt{3}-(-10)\end{pmatrix}$ Then either $\begin{pmatrix}4\sqrt{3}-\lambda\\6-\lambda\sqrt{3}\end{pmatrix}\bullet\begin{pmatrix}1\\\sqrt{3}\end{pmatrix} = 4\sqrt{3}-\lambda+6\sqrt{3}-3\lambda = 0 \Rightarrow \lambda = ...\left\{\frac{5}{2}\sqrt{3}\right\}$ $r = \begin{pmatrix}0\\-10\end{pmatrix} + \frac{5}{2}\sqrt{3}\begin{pmatrix}1\\\sqrt{3}\end{pmatrix} = ...$ Or $XW^2 = \left(4\sqrt{3}-\lambda\right)^2 + \left(6-\lambda\sqrt{3}\right)^2 = 48-8\lambda\sqrt{3}+\lambda^2+36-12\lambda\sqrt{3}+3\lambda^2$ $xw^2 = 84-20\lambda\sqrt{3}+4\lambda^2$ leading to $\frac{\mathrm{d}\left(XW^2\right)}{\mathrm{d}\lambda} = -20\sqrt{3}+8\lambda = 0 \Rightarrow \lambda = ...$	M1	3.1a
	Finds the length WX $\sqrt{\left(4\sqrt{3}-\frac{5\sqrt{3}}{2}\right)^2 + \left(-4--\frac{5}{2}\right)^2}$ Or $XW = \sqrt{\left(4\sqrt{3}-'\frac{5}{2}\sqrt{3}'\right)^2 + \left(6-'\frac{5}{2}\sqrt{3}'\sqrt{3}\right)^2}$	M1	1.1b
	So min distance is 3	A1	1.1b
		(3)	
			(10 marks)

Notes:

(a)

B1: Correct modulus

M1: Attempts the argument. Allow for $\arctan\left(\frac{\pm 4}{\pm 4\sqrt{3}}\right)$ or equivalents using the modulus (may be in wrong quadrant for this mark).

A1: Correct argument $-\frac{\pi}{6}$ (must be in fourth quadrant but accept $\frac{11\pi}{6}$ or other difference of 2π for this mark).

A1: Correct expression found for w, in the correct form, must have positive r= 8 and $\theta = -\frac{\pi}{6}$.

Note: using degrees B1 M1 A0 A0

(b)(i)&(ii)

B1: w plotted in correct quadrant with either the correct coordinate clearly seen or above the line $y = -x$

M1: Half line drawn starting on the imaginary axis away from O with positive gradient (need not be labelled)

A1: Sketch on **one diagram**– both previous marks must have been scored and the half line should pass between O and w starting from a point on the imaginary axis below w. (You may assume it starts at -10i unless otherwise stated by the candidate)

Note: If candidates draw the loci on separate diagrams the maximum they can score is B1 M1 A0

(c)

M1: Formulates a correct strategy to find the shortest distance, e.g. uses right angle OXA where X is where the lines meet and proceeds at least as far as OX.

M1: Full method to achieve the shortest distance, e.g. for $WX = OW - OX$.

A1: cao shortest distance is 3

Alternative 1:

M1: Uses a correct method to find the equation of the line from O to w, $y = -\frac{1}{\sqrt{3}}x$ and the equation of the half line $y = \sqrt{3}x - 10$, solves to find the point of intersection $X\left(\frac{5\sqrt{3}}{2}, -\frac{5}{2}\right)$

If the incorrect gradient(s) is used with no valid method seen this is M0

M1: Finds the length $WX = \sqrt{\left(\text{their}\frac{5\sqrt{3}}{2} - 4\sqrt{3}\right)^2 + \left(\text{their} -\frac{5}{2} - -4\right)^2} = ...$ condone a sign slip in the brackets.

A1: cao shortest distance is 3

Alternative 2:

M1: Uses a correct method to find the length AW and a correct method to find the angle between the horizontal and the line AW

M1: Finds the length of $WX = \text{their}\sqrt{84} \times \sin\left(\frac{\pi}{3} - \text{their } 0.7137\right) = ...$

A1: cao shortest distance is 3

Alternative 3

M1: Finds the vector equation of the half line, then XW.

Then either: Sets dot product XW and the line = 0 and solves for λ. Substitutes their λ into the equation of the half line to find the point of intersection.

Or finds the length of XW and differentiates, set = 0 and solve for λ

M1: Finds the length $WX = \sqrt{\left(\text{their}\frac{5\sqrt{3}}{2} - 4\sqrt{3}\right)^2 + \left(\text{their} -\frac{5}{2} - -4\right)^2} = ...$ condone a sign slip in the brackets.

Or substitutes their value for λ into the length of *(d)*

A1: cao shortest distance is 3