Question	Scheme	Marks	AOs
3(a)	Coordinates of Q are $(8,-3,2)$	B1	2.2a
		(1)	
(b)	Coordinates of R are $\begin{pmatrix} \cos 120° & 0 & \sin 120° \\ 0 & 1 & 0 \\ -\sin 120° & 0 & \cos 120° \end{pmatrix}\begin{pmatrix} 8 \\ 3 \\ 2 \end{pmatrix} = \ldots$ or $\begin{pmatrix} -0.5 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & -0.5 \end{pmatrix}\begin{pmatrix} 8 \\ 3 \\ 2 \end{pmatrix} = \ldots$	M1	1.1a
	So R is $\left(-4+\sqrt{3},\ 3, -4\sqrt{3}-1\right)$	A1	1.1b
		(2)	
(c)	Finds the distance $PR = \sqrt{\left(8-'\left(-4+\sqrt{3}\right)'\right)^2 + (3-'3')^2 + \left(2-'\left(-4\sqrt{3}-1\right)'\right)^2}$ Alternatively finds their $\overrightarrow{PR}$ or their $\overrightarrow{RP}$ then applies length of a vector formula. $\sqrt{\left(12-\sqrt{3}\right)^2 + \left(3+4\sqrt{3}\right)^2}$ or $\sqrt{\left(-12+\sqrt{3}\right)^2 + \left(-3-4\sqrt{3}\right)^2}$	M1	2.1
	$= \sqrt{204} \quad \left(= 2\sqrt{51}\right)$ cso	A1	1.1b
		(2)	
(d)	$\overrightarrow{PR}.\overrightarrow{PQ} = \left(-12+\sqrt{3}, 0, -3-4\sqrt{3}\right).(0,-6,0) = 0$ hence perpendicular	B1ft	1.1b
		(1)	
(e)	PQ is perpendicular to PR so Area $= \frac{1}{2} \times PQ \times PR$	M1	1.1b
	$= \frac{1}{2} \times 6 \times \sqrt{204} = 6\sqrt{51}$ cso	A1	1.1b
		(2)	
		(8 marks)	

Notes:

(a)

B1: Coordinates of Q correctly stated, accept as a column vector.

(b)

M1: Correct attempt to find coordinates of R using the given matrix with $\theta = 120$. Must be multiplying in the correct way round. With no working two correct values or (-2.27, 3, -7.93) implies this mark.

A1: Correct exact coordinates as shown in scheme. Accept as a column vector. Cos 120 and sin 120 must have been evaluated.

(c)

M1: Applies the distance formula with the coordinates of P and their R. Alternatively finds the vector $\overrightarrow{PR}$ or $\overrightarrow{RP}$ then applies length of a vector formula.

A1: Correct answer following correct coordinates of R, must be a surd but need not be fully simplified.

(d)

B1ft: Shows the dot product is zero between the vectors $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ and draws the conclusion perpendicular. Accept with $\pm$ vectors for each. Follow through as long as the vectors are of the correct form, so $\overrightarrow{PR} = \begin{pmatrix} a \\ 0 \\ b \end{pmatrix}$ and $\overrightarrow{PQ} = \begin{pmatrix} 0 \\ c \\ 0 \end{pmatrix}$

Note They could state if vectors $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ are perpendicular then $\overrightarrow{PR}.\overrightarrow{PQ} = 0$ then shows $\overrightarrow{PR}.\overrightarrow{PQ} = 0$ this is B1

(e)

M1: Correct method for the area of the triangle, follow through on their coordinates of R and Q. May see longer methods if they do not realise the triangle is right angled.

A1: For $6\sqrt{51}$ cso following correct coordinates of R

<u>Alternative 1</u>

M1 Complete method to find the correct area

Finding all the lengths $|PQ| = 6, \quad |PR| = \sqrt{240} = 4\sqrt{15}, \quad |QR| = \sqrt{204} = 2\sqrt{51}$

Use cosine rule to find an angle e.g. $\cos PRQ = \dfrac{240 + 204 - 36}{2 \times \sqrt{240} \times \sqrt{204}} = \dfrac{\sqrt{85}}{10}$

leading to $PRQ = 22.7...$ or $\sin PRQ = \sqrt{1 - \left(\dfrac{\sqrt{85}}{10}\right)^2} = ... \left\{\dfrac{\sqrt{15}}{10}\right\}$

Uses the area of the triangle $= \dfrac{1}{2} \times \sqrt{240} \times \sqrt{204} \times \dfrac{\sqrt{15}}{10}$ or $= \dfrac{1}{2} \times \sqrt{240} \times \sqrt{204} \times \sin 22.8$

A1: For $6\sqrt{51}$

<u>Alternative 2</u>

M1: Uses $\dfrac{1}{2}|\mathbf{a} \times \mathbf{b}|$ to find the required area

e.g. $QP = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}$ $RP = \begin{pmatrix} 12 - \sqrt{3} \\ 0 \\ 3 + 4\sqrt{3} \end{pmatrix}$ cross product

$$\begin{vmatrix} 0 & 6 & 0 \\ 12 - \sqrt{3} & 0 & 3 + 4\sqrt{3} \end{vmatrix} = -6\left(12 - \sqrt{3}\right)\mathbf{i} + 6\left(3 + 4\sqrt{3}\right)\mathbf{k}$$

$$\text{Area } = \frac{1}{2}\sqrt{\left(-6\left(12 - \sqrt{3}\right)^2 + \left(6\left(3 + 4\sqrt{3}\right)^2\right)\right)} = \frac{1}{2}\sqrt{7344}$$

A1: For $6\sqrt{51}$