

4(i)	$\sum \alpha_i = -\frac{5}{3} \text{ and } \sum \alpha_i \alpha_j = 0$ <p>This mark can be awarded if seen in part (ii) or part (iii)</p>	B1	3.1a
	<p>So $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = (\alpha + \beta + \gamma + \delta)^2 - 2\left(\sum \alpha_i \alpha_j\right) = \dots$</p>	M1	1.1b
	$= \frac{25}{9} - 2 \times 0 = \frac{25}{9}$	A1	1.1b
		(3)	
(ii)	$\sum \alpha_i \alpha_j \alpha_k = \frac{7}{3} \text{ and } \prod \alpha_i = 2 \text{ or for } x = \frac{2}{w} \text{ used in equation}$ <p>This mark can be awarded if seen in part (i) or part (iii)</p>	B1	2.2a
	<p>So $2\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}\right) = 2 \times \frac{\sum \alpha_i \alpha_j \alpha_k}{\alpha \beta \gamma \delta} = 2 \times \frac{\frac{7}{3}}{\frac{6}{3}}$ or for</p>	M1	1.1b
	$3\left(\frac{16}{w^4}\right) + 5\left(\frac{8}{w^3}\right) - 7\left(\frac{2}{w}\right) + 6 = 0 \Rightarrow 6w^4 - 14w^3 + \dots = 0 \text{ leading to } \frac{14}{6}$		
	$\left(= 2 \times \frac{7/3}{2}\right) \left(= \frac{14}{6}\right) = \frac{7}{3}$	A1	1.1b
		(3)	
(iii)	$(3-\alpha)(3-\beta)(3-\gamma)(3-\delta) = \dots \text{ expands all four brackets}$ <p>Or equation with these roots is $3(3-x)^4 + 5(3-x)^3 - 7(3-x) + 6 = 0$</p>	M1	3.1a
	$= 81 - 27\left(\sum \alpha_i\right) + 9\left(\sum \alpha_i \alpha_j\right) - 3\left(\sum \alpha_i \alpha_j \alpha_k\right) + \prod \alpha_i$ $= 81 - 27\left(-\frac{5}{3}\right) + 9(0) - 3\left(\frac{7}{3}\right) + 2$ <p>Or expands to fourth power and constant terms and attempts product of roots $3x^4 + \dots + 3 \times 3^4 + 5 \times 3^3 - 7 \times 3 + 6 \rightarrow \prod \alpha_i = \frac{"363"}{3}$</p>	dM1	1.1b
	$= 121$	A1	1.1b
		(3)	

(9 marks)

Notes:

(i)

B1: Correct sum and pair sum of roots seen or implied. Must realise the pair sum is zero.

Note: These values can be seen anywhere in the candidate's solution

M1: Uses correct expression for the sum of squares.

A1: $\frac{25}{9}$. Allow this mark from incorrect sign on sum of squares (but they will score B0 if the sign is incorrect).

(ii)

B1: Correct triple sum and product of roots seen or implied. May be stated in (i). Alternatively, this may be scored for sight of $x = \frac{2}{w}$ used as a transformation in the equation.

Note: These values can be seen anywhere in the candidate's solution

M1: Substitutes their values into $2 \times \frac{\sum \alpha_i \alpha_j \alpha_k}{\alpha \beta \gamma \delta} = \dots$. In the alternative it is for rearranging the equation to a quartic in w and uses to find the sum of the roots.

A1: $\frac{7}{3}$ Allow this mark from incorrect sign of both triple sum and product (but they will score B0 if the sign is incorrect).

(iii)

M1: A correct method to find the value used – may recognise structure as scheme, may expand the expression in stages, or may attempt to use a linear transformation $(3 - x)$ or e.g. $(3 - w)$ in original equation. Condone slips as long as the intention is clear.

dM1: Dependent on previous method mark. Uses at least 2 values of their sum of roots etc. in their expression. If using a linear shift this is for expanding to find the coefficient of x^4 and constant term and attempts product of roots by dividing the constant term by the coefficient of x^4 .

A1: 121.