

Question	Scheme	Marks	AOs
5(a)	$\sum_{r=1}^n (3r^2 - 17r - 25) = 3 \times \frac{n}{6}(n+1)(2n+1) - 17 \times \frac{1}{2}n(n+1) - \dots$	M1	1.1b
	$= 3 \times \frac{n}{6}(n+1)(2n+1) - 17 \times \frac{1}{2}n(n+1) - 25n$	A1	1.1b
	$= n \left(\frac{1}{2}(2n^2 + 3n + 1) - \frac{17}{2}(n+1) - 25 \right)$ or $= \frac{n}{2} \left((2n^2 + 3n + 1) - 17(n+1) - 50 \right)$	M1	1.1b
	$= n(n^2 - 7n - 33) \text{ cso (so } A = 7 \text{ and } B = 33)$	A1 cso	2.1
		(4)	
(b)	$\sum_{r=1}^{3k} r \tan(60r)^\circ$ $= \tan(60)^\circ + 2 \tan(120)^\circ + 3 \tan(180)^\circ + 4 \tan(240)^\circ + 5 \tan(300)^\circ$ $+ 6 \tan(360)^\circ + \dots$ $= (\sqrt{3} - 2\sqrt{3} + 0) + (4\sqrt{3} - 5\sqrt{3} + 0) + \dots$	M1	3.1a
	Since tan has period 180° we see $\tan(60r)^\circ$ repeats every three terms and each group of three terms results in $-\sqrt{3}$ as a sum , so with k groups of terms the sum is $-k\sqrt{3}$	A1	2.4
		(2)	
(c)	$\sum_{r=5}^n (3r^2 - 17r - 25) = \sum_{r=1}^n (3r^2 - 17r - 25) - \sum_{r=1}^4 (3r^2 - 17r - 25)$	M1	1.1b
	$= n(n^2 - 7n - 33) - 4(4^2 - 7 \times 4 - 33)$ $(= n(n^2 - 7n - 33) + 180)$	A1	1.1b
	$\sum_{r=6}^{3n} r \tan(60r)^\circ = -n\sqrt{3} + 2\sqrt{3} \text{ (allow for } -n\sqrt{3} - -2\sqrt{3} \text{)}$	B1	2.2a
	$\Rightarrow n(n^2 - 7n - 33) + 180 = 15[-n\sqrt{3} + 2\sqrt{3}]^2$ $\Rightarrow n^3 - 7n^2 - 33n + 180 = 15(3n^2 - 12n + 12)$ $\Rightarrow n^3 - 52n^2 + 147n = 0$	M1	3.1a
	$\Rightarrow n^3 - 52n^2 + 147n = 0 \Rightarrow n = \dots$	M1	1.1b
	But need $n > 5$ for sums to be valid, so $n = 49$ (allow if $n = 0$ also given but $n = 3$ must be rejected).	A1	2.3
	(6)		
(12 marks)			

Notes:

(a)

M1: Applies the formulas for sum of integers and sum of squares of integers to the summation.

A1: Correct unsimplified expression for the sum, including the $25n$

M1: Expands and factors out the n or $\frac{1}{2}n$

A1: Correct proof, no errors seen.

(b)

M1: Writes out first few terms of the sum, at least 3, and identifies the repeating pattern, e.g. through bracketed terms or stating sum repeat every three terms oe.

A1: Correct explanation identifying $-\sqrt{3}$ is the sum of each group of three terms, so with k lots of three terms the sum is $-k\sqrt{3}$

(c)

M1: Applies formula from (a) to left-hand side as a difference of two summations with either 4 or 5 as the limit on the second sum.

A1: Correct expression for the left-hand side in terms of n

B1: Correct expression for the sum on the right-hand side, allow if it arises from lower limit 6 used instead of 5 as the 6th term is zero. May subtract the first few terms directly from the work in (b).

M1: Both sides expanded and terms gathered to reach a simplified cubic equation for n with no other unknowns (may not have factor of n if errors made, which is fine for the method mark).

This mark is not dependent on any previous marks and can be awarded as long as there is an

attempt at both sides of the equation and an attempt at squaring their $\sum_{r=6}^{3n} r \tan(60r)^\circ$.

If divides through by n this mark is awarded for a 3TQ

M1: Solves their cubic equation, which may be via calculator (so may need to check values). They may divide by n and solve a quadratic. Condone decimal roots truncated or rounded

A1: Selects the correct value of n to give 49 as the only non-trivial answer. The value 3 must be rejected as summation on left undefined for this value, but accept if 0 and 49 are given (since both sides evaluate to 0 for $n = 0$ depending on one's interpretation of summations).