Question	Scheme	Marks	AOs
6(a)	Need **k** component to be zero at ground, so $0.84+0.8\lambda-\lambda^2=0 \Rightarrow \lambda=...$	M1	1.1b
	$\lambda=-\frac{3}{5},\frac{7}{5}$, but $\lambda\geqslant 0$ so $\lambda=\frac{7}{5}$	A1	1.1b
		(2)	
(b)	Direction is $(9-4.6\times1.4)\mathbf{i}+15\mathbf{j}+(0.8-2\times1.4)$ $=2.56\mathbf{i}+15\mathbf{j}-2\mathbf{k}$ or $\frac{64}{25}\mathbf{i}+15\mathbf{j}-2\mathbf{k}$	B1ft	2.2a
		(1)	
(c)	Direction perpendicular to ground is $a\mathbf{k}$, so angle to perpendicular is given by $(\cos\theta)=\frac{a\mathbf{k}.(2.56\mathbf{i}+15\mathbf{j}-2\mathbf{k})}{a\times\lvert 2.56\mathbf{i}+15\mathbf{j}-2\mathbf{k}\rvert}$ or $\frac{\begin{pmatrix}2.56\\15\\-2\end{pmatrix}\bullet\begin{pmatrix}0\\0\\a\end{pmatrix}}{\left\lvert\begin{matrix}2.56\\15\\-2\end{matrix}\right\rvert\left\lvert\begin{matrix}0\\0\\a\end{matrix}\right\rvert}$ or angle between $\begin{pmatrix}2.56\\15\\-2\end{pmatrix}$ and $\begin{pmatrix}2.56\\15\\0\end{pmatrix}$ is given by $(\cos\theta)=\frac{\begin{pmatrix}2.56\\15\\-2\end{pmatrix}\bullet\begin{pmatrix}2.56\\15\\0\end{pmatrix}}{\left\lvert\begin{matrix}2.56\\15\\-2\end{matrix}\right\rvert\left\lvert\begin{matrix}2.56\\15\\0\end{matrix}\right\rvert}$	M1	1.1b
	$=\frac{-2}{\sqrt{2.56^2+15^2+(-2)^2}}(=-0.130\ldots)$ Or $=\frac{231.5536}{\sqrt{2.56^2+15^2+(-2)^2}\sqrt{2.56^2+15^2+(0)^2}}=0.991...$	M1	1.1b
	$90°-\arccos('-0.130...')=-7.48...$ or $\arccos(0.991...)$	ddM1	3.1b
	So the tennis ball hits ground at angle of $7.5°$ (1d.p.) cao	A1	3.2a
	Alternative Finds the length of the vector in the **ij** plane $=\sqrt{2.56^2+15^2}$	M1	1.1b
	$\tan\theta=\frac{2}{\sqrt{2.56^2+15^2}}$	M1	1.1b
	$\theta=\arctan\left(\frac{2}{\sqrt{2.56^2+15^2}}\right)$ or $\theta=90-\arctan\left(\frac{\sqrt{2.56^2+15^2}}{2}\right)$	ddM1	3.1b

	So the tennis ball hits ground at angle of $7.5°$ (1d.p.)	A1	3.2a
		(4)	
(d)	In same plane as net when $\mathbf{r}.\mathbf{j} = 0$, $\begin{pmatrix} -4.1+9\lambda-2.3\lambda^2 \\ -10.25+15\lambda \\ 0.84+0.8\lambda-\lambda^2 \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ leading to $-10.25+15\lambda = 0 \Rightarrow \lambda = ...$ $\left(= \dfrac{41}{60} = 0.683333...\right)$	M1	3.1b
	So is at position $\left(-4.1+9\times\dfrac{41}{60}-2.3\left(\dfrac{41}{60}\right)^2\right)\mathbf{i}+0\mathbf{j}+\left(0.84+0.8\times\dfrac{41}{60}-\left(\dfrac{41}{60}\right)^2\right)\mathbf{k}$	M1	1.1b
	$=$ awrt $0.976\mathbf{i}+$ awrt $0.920\mathbf{k}$ or $=$ awrt $0.976\mathbf{i}+0.92\mathbf{k}$ (to 3 s.f.) or $=$ awrt $0.976\mathbf{i}+\dfrac{3311}{3600}\mathbf{k}$	A1	1.1b
		(3)	
(e)	Modelling as a line, height of net is 0.9m along its length so as **0.92 > 0.9** the ball **will** pass over the net according to the model.	B1ft	3.2a
		(1)	
(f)	Identifies a suitable feature of the model that affects the outcome And uses it to draw a compatible conclusion. For example • The ball is **not a particle** and will have **diameter/radius**, therefore it will hit the net and not pass over. • As above, but so the **ball will clip the net** but it's momentum will take it over as it is mostly above the net. • The **model** says that the ball will clear the net by 2cm which may be smaller than the balls **diameter** • The **net will not be a straight line/taut** so will not be 0.9m high, so the ball **will** have enough clearance to **pass** over the net.	M1 A1	3.2b 2.2b
		(2)	
			(13 marks)

Notes:

Accept any alternative vector notations throughout.

(a)

M1: Attempts to solve the quadratic from equating the **k** component to zero.

A1: Correct value, must select positive root, so accept 1.4 oe.

Correct answer only M1 A1

(b)

B1ft: For $(2.56, 15, -2)$ o.e or follow through $(9-4.6\times'\lambda', 15, 0.8-2\times'\lambda')$ for their λ.

(c)

M1: Recognises the angle between the perpendicular and direction vector is needed, and identifies the perpendicular as $a\mathbf{k}$ for any non-zero a (including 1), and attempts dot product

Alternatively recognises the dot product of $(2.56, 15, -2)$ and $(2.56, 15, 0)$

M1: Applies the dot product formula $\frac{a \bullet b}{|a||b|}$ correctly between *any* two vectors, but must have dot product and modulus evaluated.

ddM1: Dependent on both previous marks. A correct method to proceed to the required angle, usually $90° - \arccos('-0.130...')$ as shown in scheme but may e.g. use sin θ instead of cos θ in formula.

Alternatively is using dot product of $(2.56, 15, -2)$ and $(2.56, 15, 0)$ finds arccos(0.991…)

A1: For 7.5° cao

Alternative

M1: Finds the length of the vector in the **ij** plane.

M1: Finds the tan of any angle the

ddM1: Dependent on both previous marks. Finds the required angle

A1: For 7.5° cao

(d)

M1: Attempts to find value of λ that gives zero **j** component.

M1: Uses their value of λ in the equation of the path to find position.

A1: Correct position.

(e)

B1ft: States that 0.920 > 0.9 so according to the model the ball will pass over the net. Follow through on their **k** component and draws an appropriate conclusion. May stay the value of k > 0.92

(f)

M1: There must be some reference to the model to score this mark. See scheme for examples. It is likely to be either the ball is not a particle, or the top of the net is not a straight line. Accept references to the ball crossing a long way from the middle.

Do not accept reasons such as "there may be wind/air resistance" as these are not referencing the given model.

A1: For a reasonable conclusion based on their reference to the model.

For example

The ball is not a particle; therefore, it will not go over the net is M1A0 as not explained why – needs reference to radius/diameter