| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 7 | For $n=1$: $\begin{pmatrix}1-6\times1 & 9\times1\\ -4\times1 & 1+6\times1\end{pmatrix}=\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}=\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^1$<br>So the statement is true for $n=1$ | B1 | 2.2a |
| | **Assume** true for $\boldsymbol{n=k}$,<br>or<br>**Assume** $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^k=\begin{pmatrix}1-6k & 9k\\ -4k & 1+6k\end{pmatrix}$ | M1 | 2.5 |
| | $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^{k+1}=\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^k\times\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}$ OR $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}\times\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^k$ | M1 | 2.1 |
| | $=\begin{pmatrix}1-6k & 9k\\ -4k & 1+6k\end{pmatrix}\times\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}=\begin{pmatrix}-5+30k-36k & 9-54k+63k\\ 20k-4-24k & -36k+7+42k\end{pmatrix}$<br>OR<br>$=\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}\times\begin{pmatrix}1-6k & 9k\\ -4k & 1+6k\end{pmatrix}=\begin{pmatrix}-5+30k-36k & -45k+9+54k\\ -4+24k-28k & -36k+7+42k\end{pmatrix}$ | M1 | 1.1b |
| | Achieves from fully correct working $=\begin{pmatrix}-5-6k & 9+9k\\ -4-4k & 7+6k\end{pmatrix}$ | A1 | 1.1b |
| | $=\begin{pmatrix}1-6(k+1) & 9(k+1)\\ -4(k+1) & 1+6(k+1)\end{pmatrix}$<br>Hence the result is true for $n=k+1$. Since it is true for $n=1$, and if true for $n=k$ then true for $n=k+1$, thus by mathematical induction the result holds for all $n\in\mathbb{N}$ | A1cso | 2.4 |
| | | **(6)** | |
| | | | **(6 marks)** |

**Notes:**

**(a)**

**B1:** Shows the statement is true for $n=1$. Accept as minimum $\begin{pmatrix}1-6 & 9\\ -4 & 1+6\end{pmatrix}=\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}$

**M1:** Makes the inductive assumption, **assume** true $\boldsymbol{n=k}$**.** This may appear in the conclusion.

**M1:** A correct statement for $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^{k+1}$ in terms of $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^k$, can be either way round.

Can be implied by $\begin{pmatrix}1-6k & 9k\\ -4k & 1+6k\end{pmatrix}\times\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}$ or $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}\times\begin{pmatrix}1-6k & 9k\\ -4k & 1+6k\end{pmatrix}$

**M1:** Carries out the multiplication correctly, condone sign slips

**A1:** Correct simplified matrix **from fully correct working**

**A1:** Completes the inductive argument by showing clearly the matrix has the correct form (must have ($k$ + 1) factors in terms) or uses the result with $n=k+1$ and shows that their result is the same.

Conclusion conveying **all** three underlined points or equivalent at some point in their argument. Depends on all three M's and A marks but can be scored without the B mark as long as it is stated true for $n$ = 1