Question	Scheme	Marks	AOs
8(a)	$a = 4$	B1	3.3
		(1)	
(b)	Model A: (i) Widest point will be 4 (cm) from the base	B1	3.4
	(ii) Width at widest point is 12 (cm) $\left(2\times('a'+2)\text{ ft}\right)$	B1ft	3.4
	Model B: (i) $y = 4 + \dfrac{x^3 - 64x}{100} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3x^2 - 64}{100}$	M1	3.1b
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow x = \pm\sqrt{\dfrac{64}{3}} = \pm\dfrac{8\sqrt{3}}{3} = \pm\text{awrt}4.62$	A1	1.1b
	So max width is a distance $8 - \dfrac{8}{\sqrt{3}} = 8 - \dfrac{8\sqrt{3}}{3} \approx 3.38$ (cm) from base.	A1	3.4
	(ii) $y\big\|_{-4\ 61} = 4 + \dfrac{(-4.62...)^3 - 64(-4.62...)}{100} = ...$	dM1	3.4
	$= 5.97...$ so diameter is approximately 11.9 (cm) $\left[2a + 3.94...\text{ ft}\right]$	A1ft	3.2a
		(7)	
(c)	Model A and model B both have diameters closed to 12 Model B distance from base is closer to 3 than Model A so is more appropriate.	B1ft	3.5b
		(1)	
(d)	$V_{\mathrm{B}} = \pi\displaystyle\int_{-8}^{8} y^2\,\mathrm{d}x = \pi\int_{-8}^{8}\left(4 + \frac{x^3 - 64x}{100}\right)^2\mathrm{d}x = ...$	B1	1.1b
	$= \dfrac{\{\pi\}}{10000}\displaystyle\int_{(-8)}^{(8)} 400^2 + x^6 + 64^2x^2 + 2\left(400x^3 - 400\times 64x - 64x^4\right)\mathrm{d}x$ $= \dfrac{\{\pi\}}{10000}\displaystyle\int_{(-8)}^{(8)} 160000 + x^6 + 4096x^2 + 800x^3 - 51200x - 128x^4\ \mathrm{d}x$ $= \{\pi\}\displaystyle\int_{(-8)}^{(8)} 16 + \frac{x^6}{10000} + \frac{4096}{10000}x^2 + \frac{8}{100}x^3 - \frac{512}{100}x - \frac{128}{10000}x^4\ \mathrm{d}x$ $= \{\pi\}\displaystyle\int_{(-8)}^{(8)} 16 + \frac{x^6}{1000} + \frac{256}{625}x^2 + \frac{2}{25}x^3 - \frac{128}{25}x - \frac{8}{625}x^4\ \mathrm{d}x$ $= \{\pi\}\displaystyle\int_{(-8)}^{(8)} 16 + \frac{8x(x-8)(x+8)}{100} + \left(\frac{x(x-8)(x+8)}{100}\right)^2\ \mathrm{d}x$	M1	1.1b
	$= \dfrac{\{\pi\}}{10000}\left[160000x + \dfrac{x^7}{7} + 4096\dfrac{x^3}{3} + 800\dfrac{x^4}{4} - 51200\dfrac{x^2}{2} - 128\dfrac{x^5}{5}\right]_{(-8)}^{(8)}$	dM1	1.1b

	$=\{\pi\}\left[16x+\frac{x^7}{70000}+\frac{256}{1875}x^3+\frac{1}{50}x^4-\frac{64}{25}x^2-\frac{8}{3125}x^5\right]_{(-8)}^{(8)}$		
	$=\frac{\{\pi\}}{10000}(620583.00....--2258983.01...)\approx\frac{2879566\pi}{10000}$	M1	3.4
	$=\text{awrt}\,905\left(\text{cm}^3\right)$ cso	A1	1.1b
		(5)	
(e)	Compares their volume to 900 or compares their volume + 100 to 1 litre or 1000 and comments appropriately.	B1ft	3.5a
		(1)	
			(15 marks)

Notes:

Units not required in this question

(a)

B1: For a = 4, ignore any reference to units.

(b)

B1: Correct distance from base for Model A is 4

B1ft: Correct width at widest point. Follow through their 'a', so $2\times('a'+2)$.

M1: Attempts the derivative for Model B's equation, reduce any power by 1

A1: Sets $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and finds correct x coordinate of the stationary point (accept ±)

A1: For $8-\frac{8}{\sqrt{3}}$ or awrt 3.38 cso

dM1: Dependent on previous M mark. Uses their value of x to find the value of y. If no working shown the value of y must come from their x value.

Note using x = 4.62 give y = 2.029…

A1: Correct diameter, awrt 11.9 follow through their 'a', so $[2a+3.94...\text{ft}]$

Note: Correct answers with no working send to review

Trial and error approach

Candidates could score B1 B1 for model A however if working in integers it is unlikely that they will find the correct value for x (they are using x = – 5) not a valid method M0A0A0dM0A0

(c)

B1ft: They must have answers for all parts in (b). Accept any well-reasoned comment that follows their answers to (b) If the answers are correct, they must conclude that model B is more appropriate.

- If answers for one model are correct ish but other incorrect, or one value is clearly closer
  For example

	Distance (3)	**Diameter (12)**	**Distance (3)**	**Diameter (12)**
A	9.4	9.05	4	6
B	3.38	12.06	4.62	4.06
Conclusion	Selects B as distance/diameter closet		Select A as diameter closest	

- If distances and diameters are similar selects the model which has the most appropriate value for distance or diameter
  For example

	Distance (3)	**Diameter (12)**	**Distance (3)**	**Diameter (12)**

A	0.76	6.8	4	20
B	1.28	10.5	3.38	19.94
Conclusion	selects B as the diameter is closet		Selects B as distance is closet	

- If all values of the distances and diameters are varied any sensible reason stated for selecting a model.

(d)

B1: Applies $\pi\int_{-8}^{8} y^2 \, dx$ to the model. Must have π and correct limits, with y substituted in.

Alternatively attempts to square y first and then substitute in.

M1: Attempts to expand y^2 this can be a poor attempt but must include at least a constant and x^6 terms as long a clear attempt at y^2 (Limits not required for this mark.)

dM1: Attempts the integration, must first be rearranged to an integrable form then look for power increasing by at least 1 in at least two terms. (Limits not required for this mark.)

M1: Applies correct limits to their integral following an attempt at y^2 with at least a constant and x^6 terms.

If there is no working shown, allow this method mark if the correct answer appears from a calculator as it implies correct limits have been applied the correct way round. (So M0dM0M1 is possible.)

A1: awrt 905 cso note it must come from a fully correct solution

Note: For answers that appear from calculator B1M0dM0M1A0 is possible, the question specifies algebraic integration to be used so the integration needs to be seen to score the other marks.

(e)

B1ft: Compares their volume to 900 or compares their volume + 100 to 1 litre or 1000 and comments appropriately. Correct answer in (d) needs to conclude that it is suitable.