

Question	Scheme		Marks	AOs
4(i) (a)	$\frac{2+3i}{5+i} \times \frac{5-i}{5-i}$	$2+3i = k(1+i)(5+i) = \dots$	M1	1.1a
	$\frac{10-2i+15i+3}{25+1}$ or $\frac{13+13i}{26}$	$2+3i = k(5+i+5i-1) = \dots$	dM1	1.1b
	$\frac{1}{2}(1+i)$ cso	$2+3i = k(4+6i)$ therefore $\frac{2+3i}{5+i} = k(1+i)$ where $k = \frac{1}{2}$ cso	A1	2.1
				(3)
(i)(b)	$n = 4$			B1 2.2a
				(1)
(ii)	$ z = 3$			B1 1.2
	$\arg(z^{10}) = 10 \arg(z) = -\frac{5\pi}{3} \Rightarrow \arg(z) = \dots \left\{ -\frac{\pi}{6} \right\}$			M1 1.1b
	$\arg(z^{10}) = 10 \arg(z) = \frac{\pi}{3} \Rightarrow \arg(z) = \dots \left\{ \frac{\pi}{30} \right\}$			
	$z = 3 \left(\cos \left(-\frac{\pi}{6} \right) + i \sin \left(-\frac{\pi}{6} \right) \right) = \dots$			M1 2.1
	$z = \frac{3\sqrt{3}}{2} - \frac{3}{2}i$ or $a = \frac{3\sqrt{3}}{2}$ and $b = -\frac{3}{2}$			A1 1.1b
				(4)
	Alternative			
	$a^2 + b^2 = 9$			B1 1.2
	$10 \arg z = -\frac{5\pi}{3} \Rightarrow \arg z = -\frac{5\pi}{3} \div 10$			M1 1.1b
	Or e.g. $10 \arg(z) = \frac{\pi}{3} \Rightarrow \arg(z) = \dots \left\{ \frac{\pi}{30} \right\}$			
	Forming and solving simultaneous equations to find a value for a or b			
	$\frac{b}{a} = \arctan \left(-\frac{\pi}{6} \right) \Rightarrow \frac{b}{a} = -\frac{\sqrt{3}}{3} \Rightarrow b = -a \frac{\sqrt{3}}{3}$			M1 2.1
	or			
	$\frac{b}{a} = \arctan \frac{\pi}{30} \Rightarrow b = 0.104\dots a$			
	$z = \frac{3\sqrt{3}}{2} - \frac{3}{2}i$ or $a = \frac{3\sqrt{3}}{2}$ and $b = -\frac{3}{2}$			A1 1.1b
				(4)

(8 marks)

Notes:**(i) (a)**

M1: Selects the process $\frac{2+3i}{5+i} \times \frac{5-i}{5-i}$

dM1: Evidence of multiplying out brackets

A1: Achieves $\frac{1}{2}(1+i)$ or $\frac{13}{26}(1+i)$ with no errors cso, isw.

Note: Correct answer from no working score no marks

Note: Going from $\frac{13+13i}{26}$ and then stating $k = \frac{1}{2}$ is A0, they have not shown the form asked for

Alternative

M1: Multiplies across by $(5+i)$ and expands the brackets

dM1: Collects terms

A1: Achieves $2+3i = k(4+6i)$ and draws the conclusion that therefore $\frac{2+3i}{5+i} = k(1+i)$ where $k = \frac{1}{2}$

(i) (b)

B1: Deduces $n = 4$ only

(ii)

Note: Send to review any attempts where they are finding additional solutions such as arguments of z is $\frac{(6k-5)\pi}{30}$ For example correctly uses $\arg(z) = \frac{\pi}{30}$

B1 (M1 on ePen): $|z| = 3$ can be implied by $a^2 + b^2 = 9$ isw

M1: Uses $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$ to find $\arg(z) = -\frac{5\pi}{3} \div 10$ or $\arg(z) = \frac{\pi}{3} \div 10$

M1: Uses $z = \text{their } |z|(\cos(\text{their arg}) + i \sin(\text{their arg}))$ to find the complex number z or values for a or b .

As long as the modulus has changed.

A1: Correct complex number or values for a and b .

Alternative

B1: $a^2 + b^2 = 9$ isw

M1: Uses $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$ to find $\arg(z) = -\frac{5\pi}{3} \div 10$

M1: Uses the argument of z find an equation in a and b . Then solve simultaneously to find a value for a or b .

As long as $\sqrt{a^2 + b^2} \neq 59049$

A1: Correct complex number or values for a and b .

Note there are other correct answers

$$z_1 = \frac{3\sqrt{3}}{2} - \frac{3}{2}i$$

$$z_2 = \text{awrt } 2.98 + \text{awrt } 0.314i$$

$$z_3 = \text{awrt } 2.23 + \text{awrt } 2.01i$$

$$z_4 = \text{awrt } 0.624 + \text{awrt } 2.93i$$

$$z_5 = \text{awrt } -1.22 + \text{awrt } 2.74i$$

$$z_6 = -\frac{3\sqrt{3}}{2} + \frac{3}{2}i$$

$$z_7 = \text{awrt } -2.98 + \text{awrt } -0.314i$$

$$z_8 = \text{awrt } -2.23 + \text{awrt } -2.01i$$

$$z_9 = \text{awrt } -0.624 + \text{awrt } -2.93i$$

$$z_{10} = \text{awrt } 1.22 + \text{awrt } -2.74i$$