

Question	Scheme	Marks	AOs
6(a)	$\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = 3\{+0\} - 3$ = 0 therefore the lines are perpendicular .	M1	1.1b
		A1	2.4
		(2)	
(b)	$\mathbf{r} \bullet \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \dots \{2\}$ $x + 2y - 3z = 2$ o.e.	M1	1.1b
		A1	2.5
		(2)	
(c)	$3 + 2(1) - 3(1) = 2$ (therefore lies on the plane)	B1	1.1b
		(1)	
(d)	$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ or $\begin{pmatrix} p \\ q \\ r \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$ leading to $p = 2 - \mu$, $q = 3 - 2\mu$, $r = 2 + 3\mu$	M1	3.1a
	$(p-3)^2 + (q-1)^2 + (r-1)^2 = (2\sqrt{5})^2$		
	$((2+\mu)-3)^2 + ((3+2\mu)-1)^2 + ((2-3\mu)-1)^2 = (2\sqrt{5})^2$		
	$(-1+\mu)^2 + (2+2\mu)^2 + (1-3\mu)^2 = 20$ or	M1	3.1a
	$(p-3)^2 + (q-1)^2 + (r-1)^2 = (2\sqrt{5})^2$		
	$((2-\mu)-3)^2 + ((3-2\mu)-1)^2 + ((2+3\mu)-1)^2 = (2\sqrt{5})^2$		
	$(-1-\mu)^2 + (2-2\mu)^2 + (1+3\mu)^2 = 20$		
	$14\mu^2 - 14 = 0$ o.e	A1	1.1b
	Solves their quadratic $\{\mu = -1 \text{ or } \mu = 1\}$	M1	1.1b
	Uses $\mu = -1$ Using $\mu = 1$	ddM1	1.1b

$$\begin{array}{ll} p = 2 + (-1) = \dots & p = 2 - (1) = \dots \\ q = 3 + 2(-1) = \dots & \text{or} \\ r = 2 - 3(-1) = \dots & q = 3 - 2(1) = \dots \\ & r = 2 + 3(1) = \dots \end{array}$$

(1, 1, 5) only

A1 3.2a

(6)

Alternative

$$|AX| = \sqrt{(3-2)^2 + (1-3)^2 + (1-2)^2} = \sqrt{6}$$

Correctly uses Pythagoras to find the length of XB

$$|XB| = \sqrt{(2\sqrt{5})^2 - 6} = \sqrt{14}$$

Find the magnitude of the direction vector and compares to the length of XB to find a value for μ

$$\mu = -1 \text{ or } \mu = 1$$

M1 1.1b

Uses $\mu = -1$

Using $\mu = 1$

$$p = 2 + (-1) = \dots$$

$$p = 2 - (1) = \dots$$

ddM1 1.1b

$$q = 3 + 2(-1) = \dots \quad \text{or} \quad q = 3 - 2(1) = \dots$$

$$r = 2 - 3(-1) = \dots$$

$$r = 2 + 3(1) = \dots$$

(1, 1, 5) only

A1 3.2a

(6)

(11 marks)

Notes:

(a)

M1: Applies the dot product to the direction vectors. Minimum requirement is 3 – 3

A1: Shows that the dot product = **0** and concludes that the lines are **perpendicular**.

(b)

$$\mathbf{M1:} \text{ Applies } \mathbf{r} \bullet \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \dots \text{ or } \begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \dots$$

A1: Correct Cartesian equation $x + 2y - 3z = 2$ o.e.

Note: $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k} = 2$ is M1A0

(c)

B1: See scheme, no conclusion required

(d)

M1: Uses the point of intersection to find the coordinates of B as functions of a parameter

M1: Uses the distance between the point A and the point B to form an equation for their parameter only.

A1: Correct simplified quadratic equation

M1: Solves their quadratic equation to find a value for μ

ddM1: Dependent on the first two method marks. Uses any one of their values for their parameter to find the coordinates of B , it need not be the correct one.

A1: Correct coordinates for B , condone as a vector, if seen $(3, 5, -1)$ must be disregarded

Alternative

M1: Finds the length AX

M1: Uses Pythagoras to find the length of XB

NOTE the change in order of the M1 and A1

M1: Find the length of the direction vector and compares to find a value for μ

A1: A correct values for μ

ddM1: Dependent on the first two method marks. Uses any one of their values for their parameter to find the coordinates of B , it need not be the correct one.

A1: Correct coordinates for B , condone as a vector, if seen $(3, 5, -1)$ must be disregarded